Pi Math Contest Euler Division

Final Round - 2026 Solutions

Solutions

1. In a line of 80 people, the first 35 are dressed as superheroes and the last 36 are dressed for a vintage film night. How many people are in neither group?

   Answer (9): Since $35 + 36 = 71$, these two groups account for 71 people.

   So the number of people in neither group is

   $$80 - 71 = 9.$$

2. A farmer plants a row of 7 apple trees, spaced equally apart. The distance from the first tree to the last tree in the row is 42 feet. How many feet apart are two neighboring trees?

   Answer (7): With 7 trees in a row, the gaps between trees number one fewer than the trees themselves: $7 - 1 = 6$ gaps. The first-to-last distance of 56 feet is split equally across those 6 gaps, so each gap is $\frac{42}{6} = 7$ feet.

3. Evaluate $4 \times (\frac{2}{3} + \frac{1}{4} - \frac{1}{6})$.

   Answer (3):

   $$4 \times \left(\frac{2}{3} + \frac{1}{4} - \frac{1}{6}\right) = 4 \times \left(\frac{8}{12} + \frac{3}{12} - \frac{2}{12}\right) = 4 \times \frac{9}{12} = 4 \times \frac{3}{4} = 3.$$

4. Elizabeth reads 75% of a 16-page magazine. Tom reads all but 40% of a comic book with 15 pages. How many more pages does Elizabeth read than Tom?

   Answer (3): Elizabeth reads $0.75 \times 16 = 12$ pages. Tom reads $0.60 \times 15 = 9$ pages. Elizabeth reads $12 - 9 = 3$ more pages.

5. In the year 2026, Alex is 12 years old, his sister is 10, and his older brother is 17. After how many years will the sum of their ages be greater than 50 for the first time?

   Answer (4): Their total age in 2026 is:

   $$12 + 10 + 17 = 39.$$

   Each year, all three get one year older, so their total age increases by 3 each year. After 3 years, the sum of their ages will be $39 + 3 \times 3 = 48 < 50$ but after 4 years it will be $39 + 3 \times 4 = 51 > 50$, so the answer is 4.

6. Walter cuts a square into three congruent rectangular pieces and rearranges them to form a wider rectangle. The perimeter of the new, wider rectangle is 40 units. What is the side length of the original square?

   Answer (6): Let the width of each small rectangle be w. Since the length is three times the width, each has length $3w$. Placing the three rectangles side by side forms a new rectangle with:

   $$\text{Width} = w, \quad \text{Length} = 3 \times 3w = 9w$$

   Using the perimeter formula:

   $$2(w + 9w) = 40 \Rightarrow 20w = 40 \Rightarrow w = 2$$

   So, the length of each rectangle is $3w = 6$. Thus, the side length of the original square is 6 units.

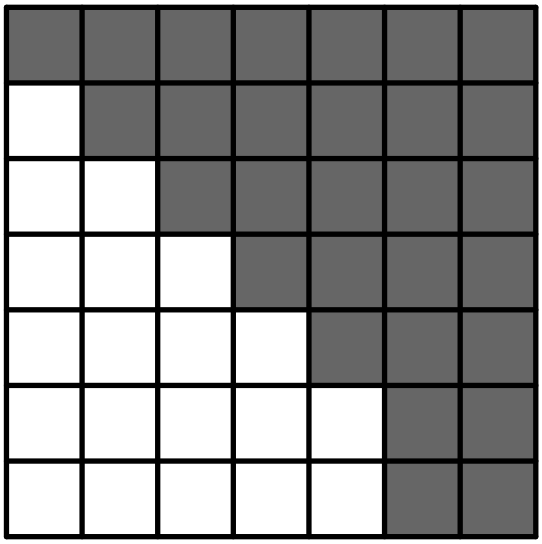
7. The average of four numbers is 6. Three of the numbers are 4, 5, and 6. What is the fourth number?

   Answer (9): Since the average of the four numbers is 6, their sum must be $4 \times 6 = 24$. The three known numbers add to $4 + 5 + 6 = 15$, so the fourth number accounts for whatever is left: $24 - 15 = 9$.

8. The product of a number and 4 is the same as the sum of the number and 12. What is the number?

   Answer (4): Let the number be n. Then $4n = n + 12$, so $3n = 12$, giving $n = 4$.

9. In a 7×7 square grid, each unit square is either black or white. How many white squares must be changed to black so that the ratio of white squares to black squares becomes $2 : 5$?



   Answer (6): The total number of unit squares in the 7×7 grid is 49. Initially, there are 29 black squares and 20 white squares.

   We are asked to make the ratio of white to black squares equal to $2 : 5$. This means that white squares should make up $\frac{2}{7}$ of the total, and black squares should make up $\frac{5}{7}$ of the total.

Calculating the target number of black squares:

$$\frac{5}{7} \times 49 = 35$$

and the target number of white squares:

$$\frac{2}{7} \times 49 = 14.$$

Currently, there are 20 white squares, but we want only 14. So we need to convert:

$$20 - 14 = 6$$

white squares into black squares.

10. Find the digit A such that the four-digit number $\overline{532A}$ leaves a remainder of 7 when divided by 9.

    Answer (6): When we divide 5320 by 9, the remainder is 1, which means that 5320 is 1 more than a multiple of 9.

    We want the completed number to leave a remainder of 7 when divided by 9, so we need to increase the number by 6 to achieve that.

    Adding 6 to 5320 gives 5326. Therefore, the missing digit must be 6.

11. A gardener waters their plants using a sequence of sprinklers. A new sprinkler activates every 10 minutes and runs for exactly 25 minutes before shutting off. If the first sprinkler turns on at $t = 0$ minutes, how many sprinklers are actively running at exactly $t = 67$ minutes?

    Answer (2): Because each sprinkler runs for 25 minutes, the sprinklers turned on at the 0, 10, 20, 30, and 40-minute marks will have already shut off before the 67-minute mark. Only the sprinklers turned on at 50 and 60 minutes will still be active, meaning exactly two sprinklers are running.

12. In the 3×3 magic square shown below, the numbers in every row, column, and diagonal sum to exactly 12.

?		5
8		
		7

Determine the value of the missing number, denoted by ?, in the upper-left corner.

Answer (1): By examining the rightmost column, the top number is 5 and the bottom number is 7. Since these already add up to 12, the middle-right cell must be 0. Moving to the middle row, the first number is 8 and the last number is 0, which means the center cell must be 4 to complete the required sum of 12. Finally, looking at the main diagonal containing the unknown top-left number, the center 4, and the bottom-right 7, we can easily determine the answer. Because 4 and 7 sum to 11, the missing top-left number must be 1.

13. A cookie recipe needs $\frac{3}{4}$ cup of flour, $\frac{1}{2}$ cup of sugar, and $\frac{1}{3}$ cup of butter per batch. A baker has 6 cups of flour, 5 cups of sugar, and 3 cups of butter. At most how many full batches can the baker make?

    Answer (8): Flour allows $6 \div \frac{3}{4} = 8$ batches. Sugar allows $5 \div \frac{1}{2} = 10$ batches. Butter allows $3 \div \frac{1}{3} = 9$ batches. The limiting ingredient is flour, so the baker can make 8 full batches.

14. Consider the following infinite sequence, formed by repeating the pattern:

    $$01234EULER56789$$

    That is, the sequence looks like:

    $$01234EULER5678901234EULER5678901234EULER56789\ldots$$

    If this pattern continues indefinitely, what is the 2026th individual character (counting both numbers and letters) in the sequence?

    Answer (0): The pattern is:

    $$01234EULER56789.$$

    Each full pattern has 15 characters.

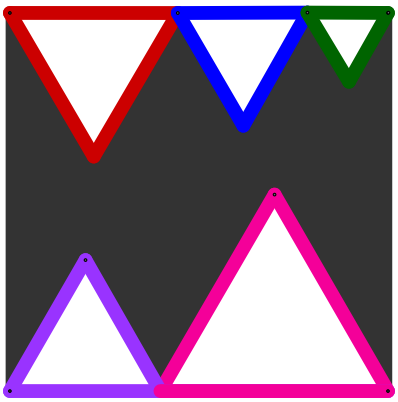
    To find the 2026th character, we divide 2026 by 15:

    $$2026 \div 15 = 135 \text{ full patterns, with 1 character left over.}$$

    So, the 2026th character is the same as the **1st character** in the pattern, which is 0.

15. In the figure below, five equilateral triangles are drawn inside a square such that their bases completely cover the top and bottom edges. This divides the square into white and black regions. If the total perimeter of the black region is 48, what is the side length of the square?

Answer (8): Observe that the top and bottom parts of the black region each measure twice the side length of the square. Consequently, the total perimeter of the black region is six times the side length of the square. Given that the perimeter of the black region is 48, the side length of the square is $\frac{48}{6} = 8$ units.



16. In the grid below, the numbers in every row, column, and diagonal form an arithmetic progression. Determine the value of the missing number, denoted by ?, in the upper-right corner.

10			**?**
	11		
		12	
22			13

Answer (1): We observe that the first column starts at 10 and ends at 22 with four numbers, implying three equal steps. The total difference is $22 - 10 = 12$, so the common difference is $12 \div 3 = 4$, giving the column: $10, 14, 18, 22$. In the second row, the first two numbers are 14 and 11, decreasing by 3. Continuing the pattern, the row is $14, 11, 8, 5$. In the last column, the second and fourth entries are 5 and 13, so the middle number is their average: $(5 + 13)/2 = 9$. Since the column increases by 4 from top to bottom (as $13 - 9 = 4$ and $9 - 5 = 4$), the top entry is $5 - 4 = 1$. Thus, the missing number is 1.

The completed grid is as follows:

10	7	4	**1**
14	11	8	5
18	15	12	9
22	19	16	13

17. A palindromic number reads the same forwards and backwards (for example, 121 or 343). How many three-digit palindromic numbers have a digit sum of exactly 15?

Answer (5): The three-digit palindromic number must have the form ABA, where:

$$2A + B = 15.$$

We find that there are 5 solutions for this: 393, 474, 555, 636, and 717.

18. Steven and Todd are playing a game called *Evens and Odds*. In each round, both players simultaneously hold out either **1** or **2** fingers.

    - If the sum of their fingers is odd, Todd wins the round.
    - If the sum is even, Steven wins the round.

    The table below shows the outcomes of 25 rounds. The rows represent Todd's choices, and the columns represent Steven's choices. Each cell indicates how many times that specific combination occurred.

Todd \Steven	**1**	**2**
1	7	6
2	4	8

    What is the positive difference between Steven's total wins and Todd's total wins?

    Answer (5): Note that Steven wins if they pick the same number of fingers and Todd wins otherwise. So, Steven wins $7 + 8 = 15$ times, whereas Todd wins $4 + 6 = 10$ times. Hence, the answer is $15 - 10 = 5$.

19. Hannah receives a weekly allowance of \$10. This year, the prices of goods increased by 25%, and her parents increased her allowance by 20%. By what percentage did the total amount of goods she can buy (her purchasing power) decrease?

    Answer (4): Hannah used to get \$10 a week and could buy 10 items if each cost \$1. After a 25% price increase, each item costs \$1.25. Her new allowance is \$12 (a 20% increase). Now, with \$12, she can buy $\$12 \div \$1.25 = 9.6$ items. This is 0.4 items less than before. So, Hannah can buy $\frac{0.4}{10} \times 100 = 4\%$ less items than before.

20. On a circular space station walkway, there are 60 evenly spaced cleaning spots. Hank the robot needs to clean every spot, but he can only move clockwise in equal steps. He must choose a step size from 2 to 9 (inclusive). Starting at a random spot, he advances by his chosen step size and cleans the spot he lands on, continuing this pattern indefinitely. Which step size should Hank choose so that he eventually lands on and cleans all 60 spots?

    Answer (7): To clean every spot, Hank's step size cannot share any common factors with 60. Since the prime factors of 60 are 2, 3, and 5, his step size cannot be a multiple of any of these. Out of the choices from 2 to 9, the only number that works is 7.

21. Determine the pattern in the first three figures to find the missing top number in Figure 4.

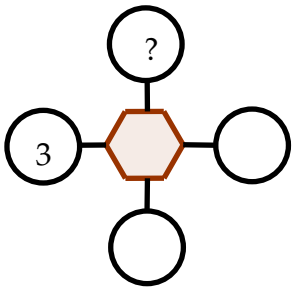


Figure 4

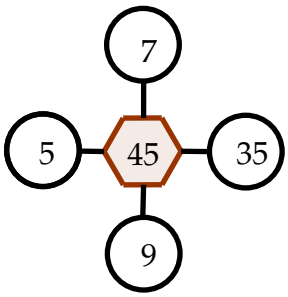


Figure 1

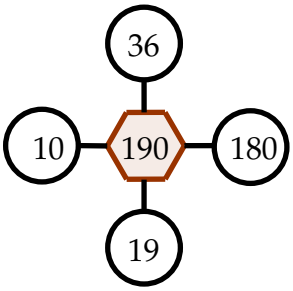


Figure 2

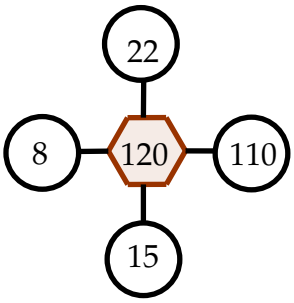


Figure 3

Answer (1): To determine the top number in Figure 4, we observe the following patterns:

- The bottom number is 1 less than twice the left number
- The middle number is the product of the left and bottom numbers
- The right number is 10 less than the middle number
- The top number is one fifth of the right number

Applying this pattern to Figure 4, we have:

- The bottom number is $2 \cdot 3 - 1 = 5$
- The middle number is $3 \cdot 5 = 15$
- The right number is $15 - 10 = 5$
- The top number is $5 \div 5 = 1$

22. Akilan and Alex are playing a game with marbles. They each start with the same number of marbles. First, Akilan gives half of his marbles to Alex. Then, Alex gives half of his marbles to Akilan. Finally, Akilan gives 2 marbles back to Alex. If they both end up with the exact same number of marbles they started with, how many marbles did Akilan have at the beginning?

    Answer (8): Let $4a$ represent the number of marbles each boy starts with. When Akilan gives half of his marbles to Alex, Akilan is left with $2a$, and Alex's pile grows to $6a$. Next, Alex gives half of his total pile, or $3a$, to Akilan. Then Akilan's new total becomes $5a$. Finally, Akilan gives 2 marbles back to Alex. Because we are told Akilan finishes the game with the exact same amount he started with, we can set up the following equation:

    $$5a - 2 = 4a.$$

    Solving this gives $a = 2$. Therefore, Akilan started with exactly $4a = 8$ marbles.

23. The product of three distinct positive integers a, b, and c is 60. Given that $a < b < c$, in how many different ways can the numbers a, b, and c be chosen?

    Answer (8): We systematically test values for a:

    If $a = 1$: then $b \times c = 60$. The valid pairs are:

    $$(1, 2, 30),\ (1, 3, 20),\ (1, 4, 15),\ (1, 5, 12),\ (1, 6, 10).$$

    If $a = 2$: then $b \times c = 30$. The valid pairs are:

    $$(2, 3, 10),\ (2, 5, 6).$$

    If $a = 3$: then $b \times c = 20$. The only valid pair is: $(3, 4, 5)$.

    Any $a \geq 4$ yields no solution because then $abc \geq 4^3 > 60$.

    Total number of ways is $5 + 2 + 1 = 8$.

24. Omer and Vera are playing a game of *Mastermind*. Vera has secretly chosen a 4-digit number with no repeated digits. Omer is trying to guess the number. After each guess, Vera tells Omer how many of the guessed digits are correct, and whether they are in the correct positions.

    Below are Omer's first four guesses and Vera's responses:

Guess 1: Omer guesses 1234.
Vera says: *"Exactly two digits are correct, but both are in the wrong positions."*

Guess 2: Omer guesses `5678`.
Vera says: *"Exactly one digit is correct, but it is in the wrong position."*

Guess 3: Omer guesses `6789`.
Vera says: *"None of these digits are in the secret number."*

Guess 4: Omer guesses `2450`.
Vera says: *"Exactly two digits are correct, and both are in the correct positions."*

Based on the information above, what is the first (leftmost) digit of Vera's secret number?

Answer (3): From Guess 3 (6789), we know that none of the digits 6, 7, 8, or 9 are in the secret number.

Between Guess 1 and Guess 2, exactly three of the digits from 1 through 8 are in the secret number (two from Guess 1, and one from Guess 2). This means the fourth digit in the secret number must be either 9 or 0. Since 9 is excluded, 0 must be in the secret number.

In Guess 2 (5678), since 6, 7, and 8 are excluded, the single correct digit must be 5.

In Guess 4 (2450), we now know the two correct digits are 5 and 0. Because they are in the correct positions, the secret number ends in 50. This also confirms that 2 and 4 are excluded.

Back in Guess 1 (1234), since 2 and 4 are excluded, the two correct digits must be 1 and 3.

We know the number is made of 1, 3, 5, and 0, and ends in 50. It cannot be 1350 because Guess 1 states its correct digits are in the wrong positions.

Therefore, the secret number is 3150, and the first digit is 3.

25. An alchemist needs to measure exactly 103 grams of a rare powder. He has an unlimited supply of 4-gram, 5-gram, and 6-gram weights. What is the difference between the largest number of weights and the smallest number of weights he can use to balance exactly 103 grams?

Answer (7): To determine the minimum number of weights, we must maximize our use of the heaviest weight, which is 6 grams. Using 15 of the 6-gram weights accounts for 90 grams. The remaining 13 grams can be balanced precisely using 1 of the 5-gram weights and 2 of the 4-gram weights. This gives us a minimum total of $2 + 1 + 15 = \mathbf{18}$ weights. The equation for this combination is:

$$2 \cdot 4 + 1 \cdot 5 + 15 \cdot 6 = 103.$$

To determine the maximum number of weights, we must maximize our use of the lightest weight, which is 4 grams. Using 23 of the 4-gram weights accounts for 92 grams. The remaining 11 grams can be balanced exactly using 1 of the 5-gram weights and 1 of the 6-gram weights. This gives us a maximum total of $23 + 1 + 1 = \mathbf{25}$ weights. The equation for this combination is:

$$23 \cdot 4 + 1 \cdot 5 + 1 \cdot 6 = 103.$$

Finally, subtracting the minimum number of weights from the maximum number of weights gives us the final difference:

$$25 - 18 = 7.$$

Note that using 17 or less weights is not possible because $17 \cdot 6 = 102 < 103$, and similarly using 26 or more weights is not possible because $26 \cdot 4 = 104 > 103$.