Pi Math Contest Fermat Division

First Round - 2026 Solutions

Solutions

1. What is the value of
$$33 - 3 \times 3 + 33 \div 3?$$

   Answer (35): Using order of operations (PEMDAS):
$$33 - 9 + 11 = 24 + 11 = 35.$$

2. The ratio of two positive numbers is 3 to 5. If the difference between the two numbers is 64, what is the smaller number?

   Answer (96): Let the numbers be $3x$ and $5x$. The difference is $2x = 64$, so $x = 32$. The smaller number is $3 \times 32 = 96$.

3. How many three-digit perfect squares are there?

   Answer (22): The smallest 3-digit perfect square is $10^2 = 100$. The largest is $31^2 = 961$. The number of integers from 10 to 31 inclusive is $31 - 10 + 1 = 22$.

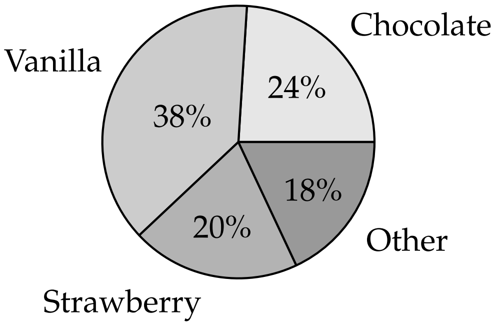
4. Eight increased by three times a number is 77. What is the number?

   Answer (23): $8 + 3x = 77 \Rightarrow 3x = 69 \Rightarrow x = 23$.

5. What is
$$\frac{2^6 - 6^2}{6-2}?$$

Answer (7): $\frac{64-36}{4} = \frac{28}{4} = 7$.

6. The 8[th] grade students at Central Middle School were surveyed on their favorite flavor of ice cream. The results are shown in the pie chart below. Given that 36 students voted chocolate as their favorite flavor, how many students voted vanilla as their favorite flavor?



Answer (57): 36 students comprise 24% of the total number of 8[th] grade students, so 3 students will comprise 2%. Since $38\% = 2\% \times 19$, the number of students who voted vanilla is $3 \times 19 = 57$.

7. The AlphaStar store is selling ramen at a price of \$2.15. If the original price was \$2.50, what percent discount is being offered?

Answer (14): The total value of the discount is 35 cents, and this is $\frac{35}{250} =$ 14% of the original price.

8. Clara purchased a ballpoint pen using three quarters and one dime, and received one nickel and two pennies in change. How many cents did the pen cost?

Answer (78): The value of three quarters and one dime is $3 \times 25 + 10 = 85$ cents. Clara received $1 \times 5 + 2 = 7$ cents in change, so the pen cost $85 - 7 = 78$ cents.

9. If $\dfrac{1}{6}+\dfrac{1}{10}=\dfrac{1}{5}+\dfrac{1}{n}$, what is the value of n?

   Answer (15): Rewrite the fractions with a common denominator of 30: $\frac{5}{30}+\frac{3}{30}=\frac{6}{30}+\frac{1}{n}$. Then $\frac{1}{n}=\frac{5}{30}+\frac{3}{30}-\frac{6}{30}=\frac{2}{30}=\frac{1}{15}$, so $n=15$.

10. The mean height of six students is 64 inches. Given that five of the students' heights are 58, 61, 64, 65, and 65 inches, what is the height of the sixth student, in inches?

    Answer (71): Let h be the height of the sixth student. Then $\frac{58+61+64+65+65+h}{6}=64 \implies 313+h=384$. Then $h=71$, so the sixth student is 71 inches tall.

    Alternate Solution: Compared to the mean height, the five students' heights are -6, -3, 0, 1, and 1 inches more than the mean. The sum of these will be $-6-3+0+1+1=-7$ inches more than 64×5, or equivalently, 7 inches less than the quantity 64×5. The sixth student must be 7 inches taller than the mean, or 71 inches tall.

11. What is the value of $\dfrac{2^{26}\times 4^{15}}{8^{17}}$?

    Answer (32): Rewrite the exponents with a common base of 2:

$$\begin{aligned}\frac{2^{26}\times 4^{15}}{8^{17}} &= \frac{2^{26}\times 2^{30}}{2^{51}}\\ &= \frac{2^{56}}{2^{51}}\\ &= 2^{56-51}\\ &= 2^5 = 32.\end{aligned}$$

12. Each interior angle of a regular polygon measures 172°. How many sides does the polygon have?

    Answer (45): We recall that the sum of the exterior angle measures in a convex polygon is 360°. Each exterior angle in this polygon measures $180^\circ-172^\circ=8^\circ$, so the number of vertices, as well as the number of sides, is $360 \div 8 = 45$.

13. The formula for converting temperatures from degrees Fahrenheit ($°F$) to degrees Celsius ($°C$) is $C = \dfrac{5}{9}(F - 32)$. The low and high temperatures in Phoenix, AZ on July 8, 2024 were 91°F and 118°F, respectively. What is the difference, in degrees Celsius, between the high and the low temperatures that day?

    Answer (15): We convert 91°F and 118°F to degrees Celsius:

$$\frac{5}{9}(91 - 32) = 32.\overline{7}\ °\text{C}$$
$$\frac{5}{9}(118 - 32) = 47.\overline{7}\ °\text{C}$$

    The difference between these two temperatures is $47.\overline{7} - 32.\overline{7} = 15$ degrees Celsius.

    Alternate Solution: Because the relationship between degrees Fahrenheit and degrees Celsius is linear, we may take the absolute difference between 91°F and 118°F, then multiply this difference by $\frac{5}{9}$. This implies that the difference, in degrees Celsius, is $\frac{5}{9}(118 - 91) = 15$ degrees Celsius.

14. A basketball conference has two divisions, each with six teams. During the season, each team plays every other team in its own division twice and each team in the other division once. How many total games are played in the conference during the season?

    Answer (96): In each division of 6 teams, they play each other twice: $\binom{6}{2} \times 2 = 30$ games. For two divisions, that's 60 games. Inter-division games: 6 teams $\times$ 6 teams = 36 games. Total is $60 + 36 = 96$.

15. Stephen was asked to type $2963 + 48715$ on a calculator. He accidentally left out two digits, and obtained a sum of 5164 instead of the correct sum of 51,678. What is the product of the two digits that Stephen left out?

    Answer (30): First, we observe that if both omitted digits are within the first number (2963), then Stephen's sum would be a 5-digit number, which is a contradiction. Additionally, if both omitted digits are within the second number (48715), then Stephen's sum would be at most $2963 + 875 < 5108$. Therefore, he left out one digit from the number 2963 and one digit from the number 48715.

Next, notice that the units digit of Stephen's sum, 4, does not match the units digit of the correct sum. This means that he must have omitted at least one of the units digits 3 or 5. The only way Stephen can obtain a units digit of 4 is if he left out the digit 5, leaving the 4-digit number 4871. To find the first missing digit, we can simply subtract $5164 - 4871 = 293$, implying that he left out the digit 6 as well. The product of the two digits that Stephen left out is $6 \times 5 = 30$.

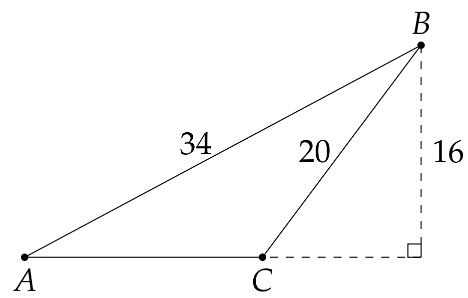
16. Kena counts backwards by 7's starting from 1000:

$$1000,\ 993,\ 986,\ 979, \ldots.$$

What is the first number that she counts that is less than 100?

Answer (97): The numbers in Kena's sequence are all 1 less than a multiple of 7 (as $1001 = 7 \times 143$). Therefore, we wish to find the largest number less than 100 which is 1 less than a multiple of 7. This number is 97, as $98 = 7 \times 14$.

17. What is the perimeter of $\triangle ABC$ in the figure below?



Answer (72): Let D be the foot of the altitude from B onto $\overline{AC}$, so that $\triangle BCD$ and $\triangle ABD$ are right triangles. By the Pythagorean Theorem on $\triangle BCD$, we have $CD = \sqrt{20^2 - 16^2} = 12$. Similarly, by the Pythagorean Theorem on $\triangle ABD$, we have $AD = \sqrt{34^2 - 16^2} = 30$. Alternatively, we may recognize that $\triangle BCD$ and $\triangle ABD$ are similar to the 3-4-5 and 8-15-17 triangles, respectively.

It follows that the length of $\overline{AC}$ is $30 - 12 = 18$, and the perimeter of $\triangle ABC$ is $34 + 20 + 18 = 72$.

18. If $a \spadesuit b = (a-2)(b-2)$, what is the value of $1 \spadesuit (2 \spadesuit (3 \spadesuit (4 \spadesuit 5)))$?

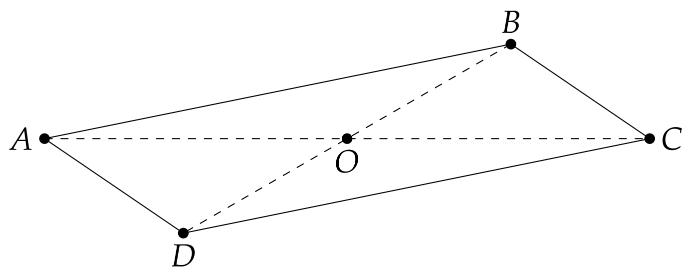
    Answer (2): We observe that $2 \spadesuit b = (2-2)(b-2) = 0$ for all numbers b. Then $1 \spadesuit (2 \spadesuit (3 \spadesuit (4 \spadesuit 5))) = 1 \spadesuit 0 = (1-2)(0-2) = 2$.

19. A pizzeria offers six different toppings, including mushrooms and olives. Joshua will select three different toppings for his personal pizza. However, he dislikes the combination of mushrooms and olives, and will not select any set of toppings which includes both of these. How many different topping combinations can Joshua choose?

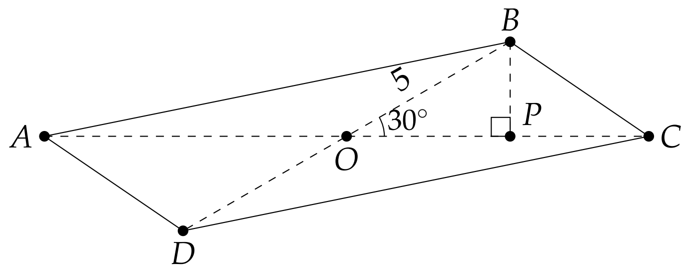
    Answer (16): There are $\binom{6}{3} = \frac{6\times5\times4}{3!} = 20$ ways to select three out of six toppings. Of these, there are 4 which include both mushrooms and olives. This leaves $20 - 4 = 16$ topping combinations which do not include both of these toppings.

20. The two diagonals of a convex quadrilateral have lengths 16 and 10, and intersect at a 30° angle. What is the area of this quadrilateral?

    Answer (40): The quadrilateral given is underspecified, as it is possible to draw multiple quadrilaterals with diagonals of lengths 16 and 10 that intersect at a 30° angle. Since the answer should not depend on which quadrilateral we draw, we can assume that the diagonals bisect each other at point O, as shown:



Assuming $AC = 16$ and $BD = 10$, we then have $AO = CO = 8$ and $BO = DO = 5$. Draw an altitude from B onto $\overline{AC}$; let P be the foot of this altitude:



Then $\triangle BOP$ is a 30-60-90 triangle with $BP = \frac{5}{2}$. Since $\overline{BP}$ is an altitude of $\triangle ABC$, which has a base of length 16, it follows that the area of $\triangle ABC$ is $\frac{1}{2} \cdot 16 \cdot \frac{5}{2} = 20$. Similarly, the area of $\triangle ADC$ is 20, and the area of $ABCD$ is $20 + 20 = 40$.

Remark: To prove that the area is 40 regardless of the shape of the quadrilateral, let $AO = a$, $BO = b$, $CO = c$, and $DO = d$, so that $a + c = 16$, $b + d = 10$, and $m\angle BOC = 30°$. Similar to the previous solution, let P be the foot of the altitude from B onto $\overline{AC}$. Then $BP = \frac{b}{2}$, and

$$\text{area of } \triangle ABC = \frac{1}{2}(a+c)\left(\frac{b}{2}\right).$$

Similarly,

$$\text{area of } \triangle ADC = \frac{1}{2}(a+c)\left(\frac{d}{2}\right).$$

Adding up these two areas, the area of $ABCD$ is

$$\frac{1}{2}(a+c)\left(\frac{b}{2}\right) + \frac{1}{2}(a+c)\left(\frac{d}{2}\right) = \frac{1}{4}(a+c)(b+d) = \frac{1}{4}(16)(10) = 40.$$

21. How many integers n with $1 \le n \le 100$ have the property that $n^2 + n$ is divisible by 6?

Answer (66): $n^2 + n = n(n+1)$. This is the product of consecutive integers, so it is always even. For it to be divisible by 6, it must also be divisible by 3. This fails only when n has remainder 1 when divided by 3. Between 1 and 100, there are 34 such numbers (1, 4, 7... 100). $100 - 34 = 66$.

22. A pair of prime numbers which differ by 4 are called *cousin primes*. For example, the pairs $(3, 7)$ and $(7, 11)$ are cousin primes. Alex writes down a pair of two 3-digit cousin primes and computes their sum. What is the largest integer that must be a factor of Alex's sum?

    Answer (6): Let p and $p + 4$ be the prime numbers, where $p \geq 100$. Note that all prime numbers are odd (except for 2), so the sum of any two 3-digit prime numbers will be even, and 2 must be a factor. Moreover, we claim that 3 is also a factor of the sum: note that a 3-digit prime number cannot be divisible by 3, so it follows that p will either be 1 or 2 more than a multiple of 3. However, if p is 2 more than a multiple of 3, then $p + 4$ will be divisible by 3, and cannot be prime. Therefore, p is 1 more than a multiple of 3 (i.e., $p = 3k + 1$ for some integer k), and $p + (p + 4) = (3k + 1) + (3k + 5) = 6k + 6 = 6(k + 1)$, proving that 6 must be a factor.

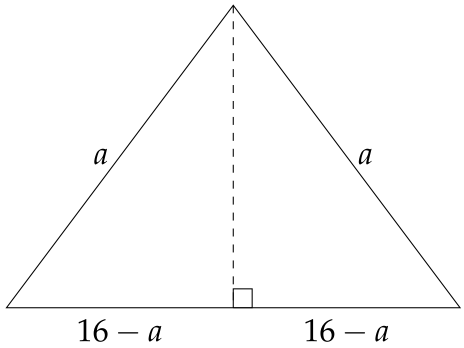
    To show that no larger integer must be a factor, consider the pairs $(103, 107)$ and $(109, 113)$, which are cousin primes. The corresponding sums are 210 and 222, and $\gcd(210, 222) = 6$. This means that no larger integer must be a factor of Alex's sum, as it would be a factor of both 210 and 222. We conclude that the answer is 6.

23. Fatima repeatedly flips a fair coin. The probability that she obtains her third head before her second tail is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

    Answer (21): Fatima's sequence of coin flips must begin with either HHH, THHH, HTHH, or HHTH. These occur with probability $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$, and $\frac{1}{16}$, respectively. As these sequences are disjoint, the probability that she will obtain her third head before her second tail is $\frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16}$, and $m + n = 5 + 16 = 21$.

24. The three sides of an isosceles triangle each measure a whole number of inches, and the perimeter of the triangle is 32 inches. Rounded to the nearest square inch, what is the greatest possible area of this triangle?

Answer (49): Suppose the sides of the triangle have lengths a, a, and $32 - 2a$ inches. Note that $9 \le a \le 15$. Draw an altitude from the apex as shown:



By the Pythagorean theorem, this altitude h has length

$$\begin{aligned} h &= \sqrt{a^2 - (16-a)^2} \\ &= \sqrt{32a - 256} \\ &= 4\sqrt{2a-16}. \end{aligned}$$

Then the area of the triangle is $4(16-a)\sqrt{2a-16}$ square inches. To maximize the area, it is sufficient to maximize the square of the area, namely $(16-a)^2(2a-16)$. Note that we ignore the constant of 16, for convenience, since this does not affect the value of a which maximizes the expression.

By substituting $a = 9, 10, \ldots, 15$, we find that this expression is maximized when $a = 11$. The maximum area is

$$4(16-11)\sqrt{2(11)-16} = 20\sqrt{6}$$

square inches. Without a calculator, we note that $2.4^2 = 5.76$ and $2.5^2 = 6.25$, and that $\sqrt{6}$ is approximately 2.45. Then $20\sqrt{6}$ is approximately 49.

25. A quadrilateral has distinct integer side lengths that form a geometric sequence. What is the smallest possible perimeter of the quadrilateral?

Answer (65): Let the sides be a, ar, ar^2, ar^3. For these to be distinct integers, the ratio r must be a fraction p/q where $p > q > 1$. Testing the smallest possible values ($p = 3, q = 2$) yields a base multiplier of $a = 8$

to keep everything whole. The sides become 8, 12, 18, and 27. The largest side (27) is strictly less than the sum of the others ($8 + 12 + 18 = 38$), so it forms a valid quadrilateral. The perimeter is $8 + 12 + 18 + 27 = 65$.