Pi Math Contest Euler Division

First Round - 2026 Solutions

Solutions

1. What is $4 + 4 \div 4$?

   Answer (5): Using order of operations: $4 \div 4 = 1$, and $4 + 1 = 5$.

2. What is the difference between twice 9 and half of 18?

   Answer (9): Twice 9 is 18. Half of 18 is 9. The difference is $18 - 9 = 9$.

3. Simplify
$$\frac{2026 + 2026 + 2026}{1013}.$$

   Answer (6):
$$\frac{2026 + 2026 + 2026}{1013} = \frac{3 \times 2026}{1013} = 3 \times 2 = 6.$$

4. What is 20% of 50% of 10?

   Answer (1): 50% of 10 is 5. Then, 20% of 5 is $0.2 \times 5 = 1$.

5. What is $14 \times 16 - 13 \times 17$?

   Answer (3): $14 \times 16 = 224$. $13 \times 17 = 221$. The difference is $224 - 221 = 3$.

6. How many ways are there to choose one letter from $\{A, B, C\}$ **and** one number from $\{1, 2, 3\}$?

   Answer (9): There are 3 choices for the letter and 3 choices for the number. $3 \times 3 = 9$ combinations.

7. How many times does the digit 2 appear when writing all the prime numbers less than 100?

   Answer (3): The prime numbers less than 100 that contain the digit 2 are 2, 23, and 29. The digit 2 appears exactly 3 times.

8. On Halloween, Alice receives 12 candies, and each of her three friends receives 4 candies. They put all their candies together and then share them equally among the four of them. How many candies does each person receive?

   Answer (6): Alice has 12 candies, and her 3 friends have $3 \times 4 = 12$ candies. The total is $12 + 12 = 24$. Shared equally among the 4 of them, each gets $24 \div 4 = 6$ candies.

9. How many different prime factors does a million have?

   Answer (2): One million is $1,000,000 = 10^6$. The prime factorization is $2^6 \times 5^6$. There are exactly 2 different prime factors (2 and 5).

10. Five consecutive integers have a sum of 30. What is the smallest integer?

    Answer (4): The average of the five numbers is $30 \div 5 = 6$. Since they are consecutive, 6 is the middle number. The sequence is 4, 5, 6, 7, 8. The smallest is 4.

11. How many positive even factors does the number 28 have?

    Answer (4): The factors of 28 are 1, 2, 4, 7, 14, and 28. The even factors are 2, 4, 14, and 28. There are 4 of them.

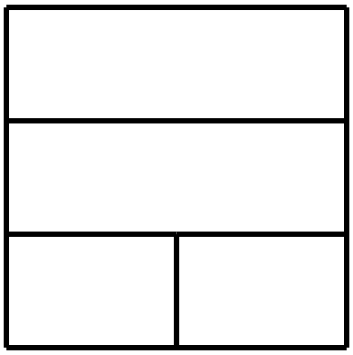
12. Let A be the area of a triangle with base 28 and height 12, and let B be the area of a rectangle with length 14 and height 12. What is $A - B$?

Answer (0): Note that $A = \frac{1}{2} \times 28 \times 12 = 14 \times 12 = B$. Therefore, $A - B = 0$.

13. Eight boxes each contain 18 candies. If the candies are divided equally among 24 students, how many candies does each student receive?

    Answer (6): There are $8 \times 18 = 144$ candies. So, each student receives $144 \div 24 = 6$.

14. How many rectangles of any size are in the figure below?



    Answer (8): Looking at the figure: There is 1 top row, 1 middle row, and 2 small rectangles in the bottom row. Counting all combinations:

    - 4 individual blocks
    - 2 combinations of two blocks (top+mid, and the two bottom ones together)
    - 1 combination of three blocks (mid+bottom row)
    - 1 large outer rectangle

    giving a total of 8 rectangles.

15. A chipmunk eats 24 acorns in 40 seconds at a constant rate. How many minutes will it take the chipmunk to eat 252 acorns at the same rate?

    Answer (7): The chipmunk eats 12 acorns in 20 seconds and 36 acorns in 1 minute. At this rate, it will take the chipmunk $\frac{252}{36} = 7$ minutes to consume 252 acorns.

16. Peter begins skip-counting at 38 and continues by adding the same number each time until he reaches 134. In total, he counts 11 numbers between 38 and 134,

excluding both endpoints. What is the skip-counting interval?

Answer (8): There are 11 numbers between 38 and 134, making 13 total terms in the sequence (12 "jumps"). The total difference is $134 - 38 = 96$. The skip interval is $96 \div 12 = 8$.

17. Samantha has \$35 and saves \$8 each week. Taylor has \$100 and spends \$5 each week. After how many weeks will they have the same amount of money?

Answer (5): Initially, the difference between Samantha's and Taylor's amounts is $100 - 35 = 65$ dollars. This difference decreases by $8 + 5 = 13$ dollars per week.

Therefore, Samantha and Taylor will have the same amount of money after $\frac{65}{13} = 5$ weeks.

18. How many integers from 1 to 31 have a units digit of 1 when squared?

Answer (7): For a square to end in 1, the original number's units digit must be 1 or 9. There are 7 such numbers between 1 and 31: 1, 9, 11, 19, 21, 29, and 31.

19. When the product

$$5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40$$

is written as a decimal, how many zeros are at the end of the number?

Answer (7): To find trailing zeros, count the number of 5s and 2s in the prime factorizations. The product has nine 5s but only seven 2s ($10 = 2 \times 5$, $20 = 2^2 \times 5$, $30 = 2 \times 15$, $40 = 2^3 \times 5$). The zeros are limited by the number of 2s, which is 7.

20. In how many ways can 17 be written as a sum of 2's, 3's, and 7's if the order does not matter? For example, $7 + 3 + 7$ and $3 + 7 + 7$ are considered the same.

Answer (6): Let's break it down into cases based on the number of 7's used in the combinations:

Case 1: Using two 7's:

$$7 + 7 + 2 + 2$$

Case 2: Using one 7:

$$7 + 3 + 3 + 2 + 2, \quad 7 + 2 + 2 + 2 + 2 + 2$$

Case 3: Using no 7's

$3+3+3+3+3+2, \quad 3+3+3+2+2+2+2, \quad 3+2+2+2+2+2+2+2$

So, there are $1+2+3=6$ ways.

21. Anthony donates 230 books to a children's hospital. When the books are distributed equally among the children, 9 books remain. Each child receives more books than the number of children in the hospital. What is the smallest number of additional books Anthony must donate so that all the books can be distributed equally with none left over?

Answer (4): $230-9=221$ books distributed evenly. The factors of 221 are 13×17. Because each child receives more books than the total number of children, there must be 13 children receiving 17 books each. The next multiple of 13 after 230 is 234. Therefore, Anthony needs $234-230=4$ more books.

22. The following sequence is given:

$$a,\ b,\ c,\ 8,\ 4,\ 7,\ 5,\ 6,\ \ldots$$

After the first three terms, each subsequent term is obtained by adding the first two of the previous three terms and subtracting the third. For example, the fourth term satisfies $a+b-c=8$, and the seventh term is $8+4-7=5$.

What is a?

Answer (2): Let T_n be the nth term. The rule is $T_n = T_{n-3}+T_{n-2}-T_{n-1}$. Working backwards:

$$c+8-4=7 \Rightarrow c=3.$$

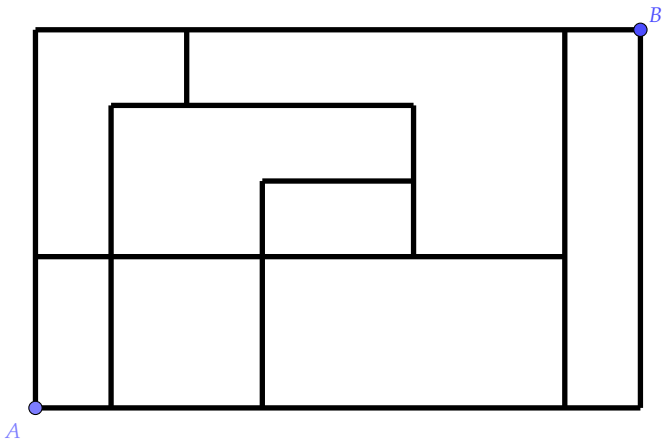
Then

$$b+3-8=4 \Rightarrow b=9.$$

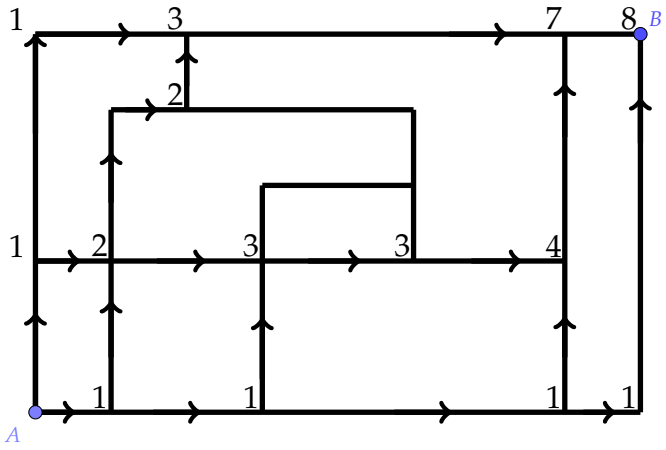
Finally,

$$a+9-3=8 \Rightarrow a=2.$$

23. In the diagram below, all angles are right angles. How many shortest paths are there from point A to point B?



Answer (8): Shortest paths consist of only upward and rightward movements. By examining each intersection point starting from point A and moving exclusively rightward and upward, we can apply the principle of addition to determine the total count of distinct paths. In this case, there are a total of 8 shortest paths leading from point A to point B.



24. A jar contains 30 candies of four flavors: strawberry, chocolate, vanilla, and raspberry.

    Any 23 candies chosen from the jar include at least one vanilla candy.

    Any 26 candies chosen from the jar include at least one strawberry candy.

Any 21 candies chosen from the jar include at least one raspberry candy.

What is the greatest possible number of chocolate candies in the jar?

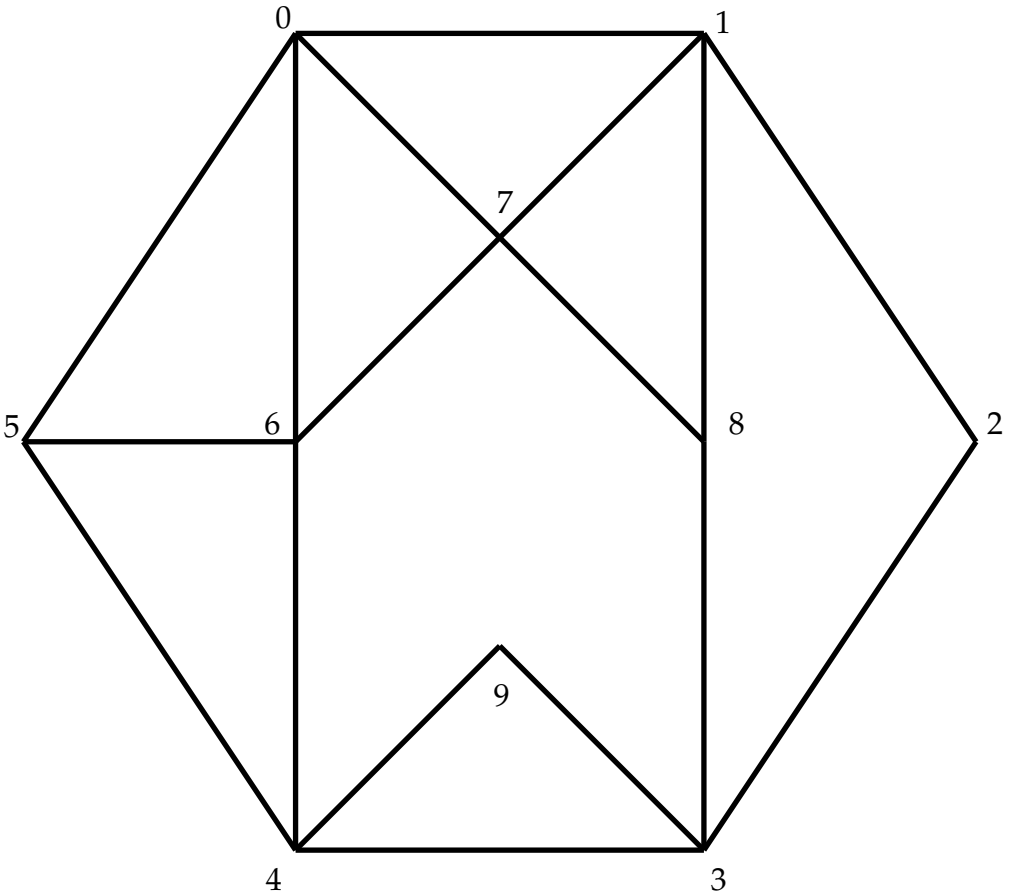
Answer (7): When 23 candies are randomly selected, at least one of them is vanilla. Therefore, the number of vanilla candies is at least 8, or otherwise we could choose 23 non-vanillay candies.

When 26 candies are selected, at least one of them is strawberry. Therefore, the number of strawberry candies is at least 5.

If 21 candies are selected, at least one of them is raspberry. Therefore, the number of raspberry candies is at least 10.

Then the sum of vanilla, strawberry, and raspberry candies is at least $8 + 5 + 10 = 23$ and the number of chocolate flavor candies is at most $30 - 23 = 7$.

25. In the following diagram, each intersection is labeled with a digit. Starting at intersection 5, we trace each edge exactly once. At which intersection does the tracing end?



Answer (8): At intersection 5, three edges converge, while at intersection 0, four edges converge. Upon inspection, we observe that all intersections have an even number of edges converging, except for intersections 5 and 8. Even-numbered intersections imply that if tracing arrives at them, it must depart. Therefore, the tracing must conclude at intersection 8.