Pi Math Contest Gauss Division

2025

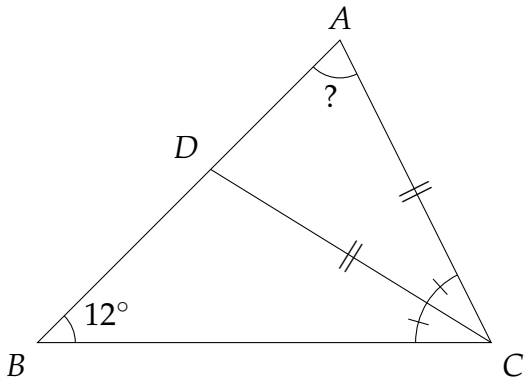
INSTRUCTIONS

1. DO **NOT** OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU!
2. This is a 25 question test. Each question has a three digit answer: 000, 001, 002, 003, . . ., 999. For example, if the answer is 1, you must bubble 001 instead of 1.
3. Mark your answer to each question on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 10 points for each correct answer, 1 point for each problem left unanswered, and 0 points for each incorrect answer.
5. Only pencils, pencil sharpeners, erasers, and blank scratch papers are allowed. All other aids, including but not limited to calculators and notes, are not allowed.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
8. After the exam, return your **Answer Form, Test Booklet and scratch papers** to your proctor.
9. Do NOT discuss any exam questions until **March 24, 2025**, after which problems and solutions will be available on the contest website.

1. What is the value of $\sqrt{21+\sqrt{13+\sqrt{7+\sqrt[3]{8}}}}$?

2. How many digits does the number $32^{11} \cdot 125^{15}$ have in its decimal representation?

3. In $\triangle ABC$ shown below, $\angle B = 12^\circ$, $\overline{CD}$ is the angle bisector of $\angle C$, and $CA = CD$. What is the degree measure of $\angle A$?



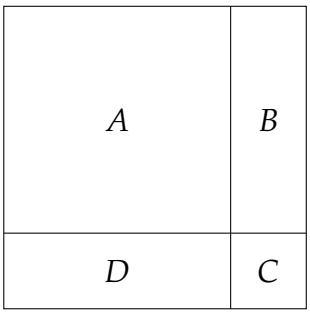
4. What is the smallest positive integer n such that $12! \times n$ is the square of an integer?

5. When a positive integer A is divided by 50, the quotient is n and the remainder is n^2 for some positive integer n. What is the maximum value of A?

6. The radius of each rear wheel of a tractor is double the radius of each front wheel. After the tractor moved 960π inches without slippage, each front wheel made 20 more revolutions than each rear wheel. What is the diameter, in inches, of each rear wheel?

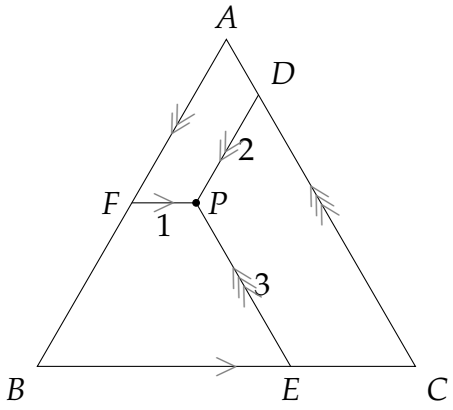
7. In a group of people, the average height of the males is 174 centimeters, the average height of the females is 164 centimeters, and the average height of all people in the group is 173.9 centimeters. What is the ratio of the number of males to the number of females in the group?

8. In the figure below, a square is divided into two squares (A and C) and two non-square rectangles (B and D). Both diagonals of B have length 16, and both diagonals of the large square have length 30. What is the sum of the areas of rectangles B and D?



9. The letters in the phrase "MAMMA MIA" are rearranged to make an eight-letter "word." In how many of these words does the first M appear before the first A? One example is MMAMAAIM.

10. In the figure below, $\triangle ABC$ is an equilateral triangle. Segments $\overline{PD}$, $\overline{PE}$ and $\overline{PF}$ are parallel to sides $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, respectively, and $PD = 2$, $PE = 3$, and $PF = 1$. What is the square of the area of $\triangle ABC$?

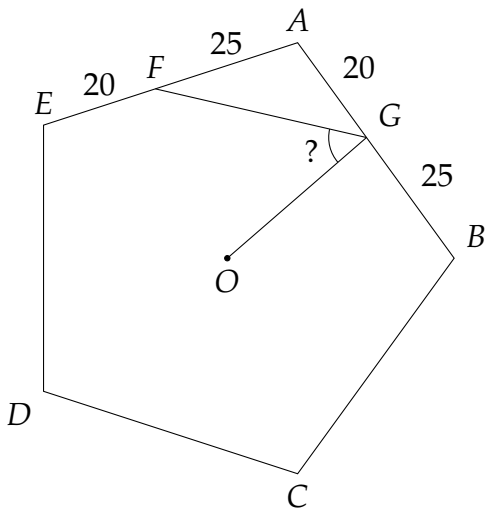


11. Alice writes a sequence of digits on the blackboard. She realizes that any positive integer less than 1000 can be obtained by erasing some of the digits on the board. What is the fewest number of digits Alice could have written?

12. How many positive 5-digit integers are there whose digits multiply to 80?

13. The positive integer N is such that the number N^2 is equal to the 4-digit number $\overline{AABB}$, where A and B are digits and $A \neq 0$. What is the value of N?

14. $ABCDE$ is a regular pentagon and O is its center. Points F and G are on $\overline{AE}$ and $\overline{AB}$, respectively, such that $AF = BG = 25$ and $EF = AG = 20$. What is the degree measure of $\angle OGF$?



15. 1000 eggs are distributed into 10 baskets, with each basket containing a different number of eggs. If the basket with the greatest number of eggs contains N eggs, what is the smallest possible value of N?

16. The following increasing sequence contains all positive integers which are equal to the sum of two distinct powers of 2:

$$3, 5, 6, 9, 10, 12, 17, \ldots.$$

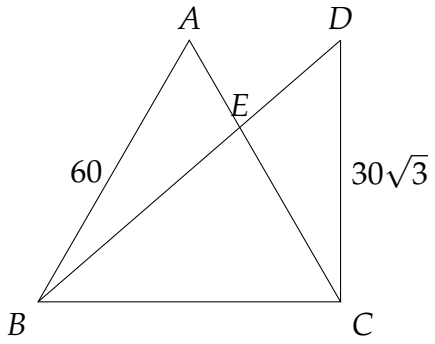
The 2025^{th} number in this sequence is equal to $2^a + 2^b$, where a and b are distinct positive integers. What is the value of ab?

17. How many positive integers less than or equal to 1000 are divisors of the number $13^{1000!} - 11^{1000!}$?

18. A positive divisor of 6^{1000} is chosen at random. If the probability that the chosen number is a multiple of 6^{91} is $\frac{m}{n}$ where m and n are relatively prime positive integers, what is $m+n$?

19. Let $P(x)$ be a quadratic polynomial such that $P(1)=2$, $P(2)=3$, and $P(3)=1$. What is the value of $P(4)^2$?

20. In the figure below, $\triangle ABC$ is an equilateral triangle, $AB=60$, $CD=30\sqrt{3}$, and $\angle BCD$ is a right angle. What is AE^2?



21. Let the sequence a_n be defined for all positive integers n by the rule

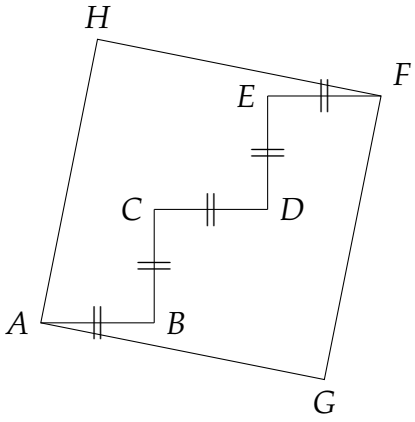
$$a_n = n^2 + n + 1.$$

Given that $a_{30}a_{31} = a_N$ for some integer N, what is the value of N?

22. In a 25-question test, each correct answer earns 4 points, leaving a question blank earns 0 points, and each incorrect answer incurs a deduction of 1 point. How many different scores are possible?

23. The numbers in the set $\{1,2,3,4,\ldots,75\}$ are divided into 15 subsets with each subset containing exactly 5 numbers. No two subsets have a number in common. What is the largest possible value of the sum of the medians of the subsets?

24. In the figure below, $AGFH$ is a square, the angles at B, C, D and E are all right angles, and $AB = BC = CD = DE = EF = 10$. What is the area of square $AGFH$?



25. Real numbers x and y satisfy the equation $x^2 - y^2 = 100$. What is the smallest possible value of the expression $(3x + y)^2$?