Pi Math Contest Fermat Division

2025 Solutions

The problems and solutions in this contest were proposed by:

Akilan Paramasivam, Isaac Li, Rachel Li, Richard Spence, and Stanley Wang.

Solutions

1. One gallon of gasoline costs \$3.85. How many dollars will Inbar pay for 20 gallons of gasoline?

   Answer (77): Inbar will pay $\$3.85 \times 20 = \$77$ for 20 gallons of gasoline.

2. Farmer John uses 36 feet of fencing to enclose a square pigpen. In square feet, what is the area of the pigpen?

   Answer (81): We are looking for the area of a square whose perimeter is 36 feet. The side length of the pigpen is $36 \text{ ft} \div 4 = 9 \text{ ft}$, so its area is $(9 \text{ ft})^2 = 81$ square feet.

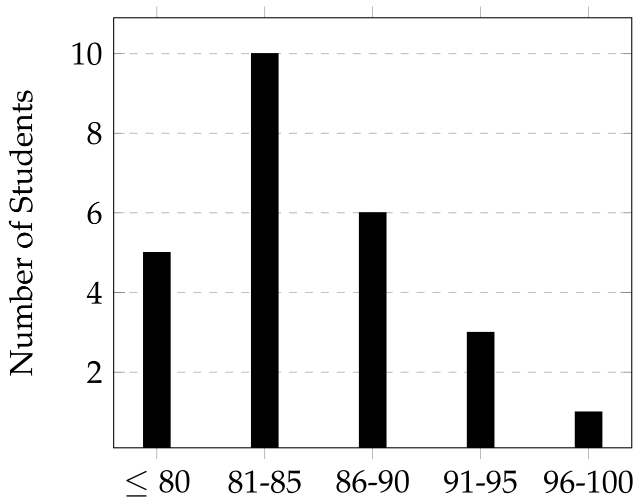
3. How many integers n satisfy the inequality $|n| < 4\pi$?

   Answer (25): The value of 4π is approximately 12.57, so the integers -12, -11, -10, $\ldots$, 11, 12 satisfy the given inequality. Altogether, there are 25 integers.

4. What is the value of $\frac{5}{3} \times \frac{6}{4} \times \frac{7}{5} \times \ldots \times \frac{23}{21} \times \frac{24}{22}$?

   Answer (46): All terms cancel except for 23×24 in the numerator and 3×4 in the denominator. The given expression equals $\frac{23 \times 24}{3 \times 4} = \frac{23 \times (2 \times 3 \times 4)}{3 \times 4} = 23 \times 2 = 46$.

5. The histogram shows the distribution of the students' quiz scores in Ms. Wood's geometry class. What percentage of the students in the class scored greater than or equal to 91 points?



   Answer (16): By reading the data values in the histogram, we see that there are $5 + 10 + 6 + 3 + 1 = 25$ students in the class, and $3 + 1 = 4$ students scored greater than or equal to 91 points. As a percentage, this equals $4 \div 25 = 16\%$.

6. The diagonal of a TV measures 65 inches, and the ratio of the length of the TV to its width is $4 : 3$. What is the length of the TV, in inches?

   Answer (52): Suppose the length and width of the TV are $4x$ and $3x$ inches, respectively. We recall the 3-4-5 Pythagorean triple; this implies that the diagonal has length $5x$ inches. We are given that the diagonal has length 65 inches, so $5x = 65 \implies x = 13$. The length of the TV is

$4x = 4 \times 13 = 52$ inches.

7. How many positive 3-digit integers are there, such as 105, whose digits add up to 6?

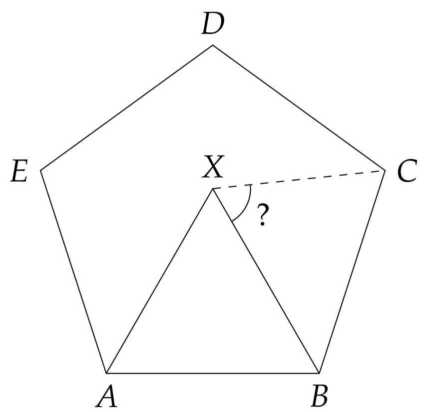
   Answer (21): We can list out all 3-digit numbers in order of their hundreds digit:

   - 105, 114, 123, 132, 141, 150
   - 204, 213, 222, 231, 240
   - 303, 312, 321, 330
   - 402, 411, 420
   - 501, 510
   - 600

   In general, if the hundreds digit is k, then the remaining two digits must add to $6-k$, in which there are $7-k$ integers. Altogether, there are $6+5+4+3+2+1 = 21$ integers.

   Alternate Solution: Suppose the digits are $\overline{abc}$. We wish to find the number of ordered triples (a, b, c) of digits where $a \geq 1$ and $a+b+c = 6$. Let $a' = a-1$, so that a' is non-negative. Then we can equivalently find the number of non-negative integer solutions to the equation $a' + b + c = 5$. Using the Stars and Bars method, there are $\binom{5+3-1}{3-1} = \binom{7}{2} = 21$ integers.

8. Equilateral triangle ABX is drawn inside regular pentagon $ABCDE$. What is the degree measure of $\angle BXC$?



Answer (66): We recall that the sum of the interior angles in an n-sided polygon is $180(n-2)^\circ$, and that each interior angle has measure $\frac{180(n-2)}{n}$ degrees. This implies that $m\angle ABC = 108^\circ$ and $m\angle ABX = 60^\circ$. Then $m\angle XBC = m\angle ABC - m\angle ABX = 108^\circ - 60^\circ = 48^\circ$. Next, we observe that $\triangle BCX$ is isosceles since $XB = BC$. Then $m\angle BXC = m\angle BCX$. The sum of the degree measures in $\triangle BCX$ is 180°, so $m\angle BXC = \frac{180^\circ - 48^\circ}{2} = 66^\circ$.

9. When $4^{14} \times 5^{20}$ is multiplied and written out, how many digits does it contain?

   Answer (23): Rewrite 4^{14} as $(2^2)^{14} = 2^{28}$. The given number equals $2^{28} \times 5^{20} = 2^8 \times (2^{20} \times 5^{20}) = 256 \times 10^{20}$. This number consists of the digits "256" followed by 20 zeros. Altogether, there are $3 + 20 = 23$ digits.

10. Alyssa has an unfair coin which lands heads with probability 60%. Expressed as a percent, if Alyssa flips this coin twice, what is the probability that she obtains exactly one head?

    Answer (48): Alyssa can either flip a head followed by a tail, or a tail followed by a head. The probability of either of these outcomes is $0.6 \times 0.4 = 0.24 = 24\%$. The probability that she obtains exactly one head is $2 \times 24\% = 48\%$.

11. What two-digit integer is equal to 6 times the sum of its digits?

    Answer (54): Suppose the digits are $\overline{ab}$, where $1 \le a \le 9$ and $0 \le b \le 9$. Then $10a + b = 6(a + b)$; simplifying yields $4a = 5b$. Since $a \ne 0$, the only possibility is $a = 5$ and $b = 4$, and the corresponding two-digit integer is 54.

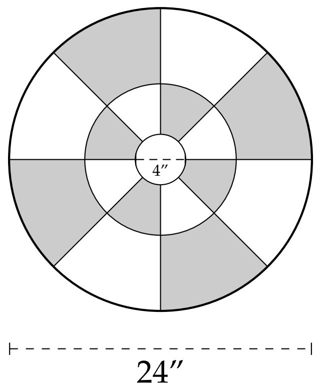
12. Five students, including Gilbert and Raymond, sit in a row of five seats. How many different ways can all five students seat themselves so that Gilbert and Raymond do **not** sit next to each other?

    Answer (72): We can manually count that there are 6 ways to choose the seats for Gilbert and Raymond so that their seats are not next to each other (seats 1 and 3, 1 and 4, 1 and 5, 2 and 4, 2 and 5, or 3 and 5). There are 2 ways to seat them at these two seats, followed by $3! = 6$ ways to seat the remaining three students. The number of ways is $6 \times 2 \times 6 = 72$.

Alternate Solution: We can use complementary counting. There are $5! = 120$ total ways to seat 5 students without restriction.

If Gilbert and Raymond sit together, then there are 4 ways to choose the two seats that they occupy, followed by 2 ways to seat Gilbert and Raymond in these two seats, and $3! = 6$ ways to seat the remaining students. There are $4 \times 2 \times 6 = 48$ ways where Gilbert and Raymond sit together. Therefore, the number of ways where they do not sit together is $120 - 48 = 72$.

13. A custom dartboard is divided into 17 regions as shown. The diameters of the dartboard and the central bullseye are 24 inches and 4 inches, respectively. If the total area of the shaded regions is $k\pi$ square inches, what is k?



Answer (70): Exactly half of the area of the dartboard which is outside the bullseye region is shaded. The dartboard has radius 12 inches, so its area is $\pi(12 \text{ in})^2 = 144\pi \text{ in}^2$. The bullseye has radius 2 inches, so its area is $\pi(2 \text{ in})^2 = 4\pi \text{ in}^2$. The area of the dartboard which is outside the bullseye is $144\pi \text{ in}^2 - 4\pi \text{ in}^2 = 140\pi \text{ in}^2$. The shaded area is half of this, or $70\pi \text{ in}^2$, so $k = 70$.

14. A function $f(x)$ satisfies the property that $f(x+y) = f(x) + f(y) - 7$ for all integers x and y. What is $f(1) + f(-1)$?

    Answer (14): By substituting $x = 1$ and $y = -1$, we obtain $f(0) = f(1) + f(-1) - 7$, so $f(1) + f(-1) = f(0) + 7$. To determine $f(0)$, we substitute $x = y = 0$ to obtain $f(0) = 2f(0) - 7 \implies f(0) = 7$. Therefore $f(1) + f(-1) = 7 + 7 = 14$.

    Remark: One example function which satisfies this property is $f(x) = x + 7$.

15. Jesse rode his bicycle along a path consisting of uphill and downhill segments. He averaged 6 miles per hour (mph) going uphill, and 15 mph going downhill. Jesse averaged 10 mph throughout his trip, and rode for a total of 90 minutes. How many minutes did Jesse spend riding downhill?

    Answer (40): Suppose Jesse rode m minutes downhill, and $90 - m$ minutes uphill. Using $d = rt$, Jesse rode a total of $d = 10\,\frac{\text{mi}}{\text{hr}} \times 1.5\,\text{hr} = 15$ miles. We can also compute the total distance in terms of m, namely $d = 6\,\frac{\text{mi}}{\text{hr}} \times \frac{90-m}{60}\,\text{hr} + 15\,\frac{\text{mi}}{\text{hr}} \times \frac{m}{60}\,\text{hr}$. Setting the two expressions for d equal, we can solve for m:

$$\begin{aligned} 6\left(\frac{90-m}{60}\right) + 15\left(\frac{m}{60}\right) &= 15 \\ 9 + \frac{9m}{60} &= 15 \\ m &= 40. \end{aligned}$$

16. Nigel has a collection of \$1, \$5, and \$20 bills in his savings jar. He has 22 bills in his savings jar, which are worth a total of \$164. How many \$5 bills does Nigel have in his savings jar?

    Answer (07): Suppose Nigel has a \$1 bills, b \$5 bills, and c \$20 bills. Then $a + b + c = 22$ and $a + 5b + 20c = 164$. Ordinarily, since there are more variables than equations, we cannot solve for a unique solution; however, we will use the fact that a, b, and c are non-negative integers.

Note that a must be 4 more than a multiple of 5, since $5b + 20c$ is necessarily a multiple of 5. Therefore, a can be 4, 9, or 14 ($a = 19$ is clearly too large, as this would imply $b + c = 3$ and the total value of the bills cannot be \$164). We consider these three possibilities:

- **Case 1:** $a = 4$. Then $b + c = 18$ and $5b + 20c = 160 \iff b + 4c = 32$. Subtracting these two equations yields $3c = 32 - 18 = 14$, so $c = \frac{14}{3}$ which is not an integer. Therefore, we can discard this case.
- **Case 2:** $a = 9$. Similarly, we obtain $b + c = 13$ and $5b + 20c = 155 \iff b + 4c = 31$. This system of two equations has a solution, $(b, c) = (7, 6)$.
- **Case 3:** $a = 14$. Similarly, we obtain $b + c = 8$ and $5b + 20c = 150 \iff b + 3c = 30$. Similar to Case 1, the value of c is not an integer, so we discard this case.

The only possibility is $(a, b, c) = (9, 7, 6)$. The problem asks for the value of b; Nigel has 7 \$5 bills.

17. Which two-digit positive integer has exactly 6 odd positive factors and 6 even positive factors?

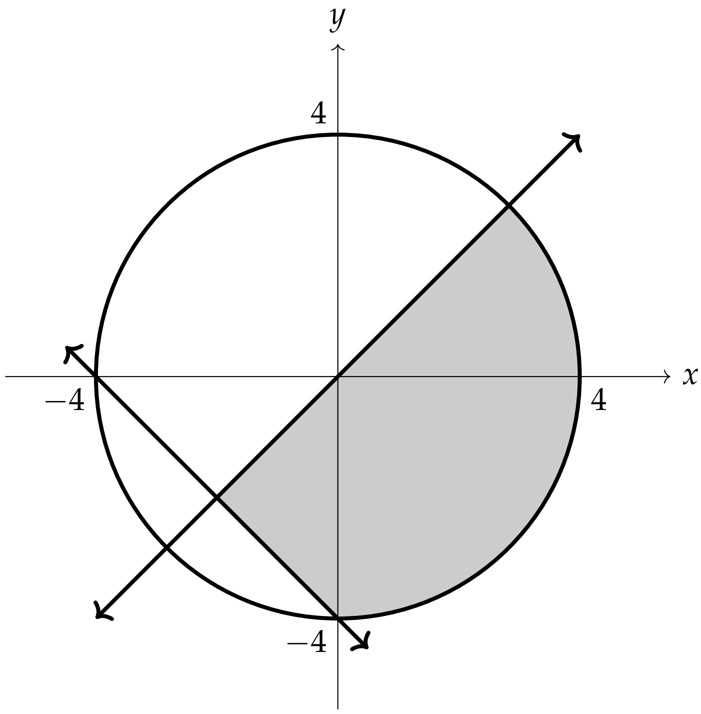
Answer (90): Let n be the desired two-digit integer, and suppose $n = 2^a \times b$ for some non-negative integers a and b, where b is odd. The odd factors of n are precisely the factors of b, so b must have 6 odd positive factors. We recall the method for finding the number of divisors of a number, by adding 1 to each of the exponents in the prime factorization and multiplying the resulting exponents. It follows that b is of the form p^5 or p^2q where p and q are odd and different prime numbers. If $b = p^5$, then the smallest possible value of b is $3^5 = 243$, which is clearly too large. We conclude $b = p^2q$, and the smallest possible value of b is $3^2 \times 5 = 45$ (the next smallest candidate value of b is $3^2 \times 7 = 63$, which we will show is also too large).

Next, in order for n to have the same number of odd and even positive factors, a must equal 1. Therefore $n = 2b$; since it is given that n is a 2-digit integer, it follows that $b < 50$. The only candidate value of b less than 50 is 45, so we conclude that $n = 2 \times 45 = 90$.

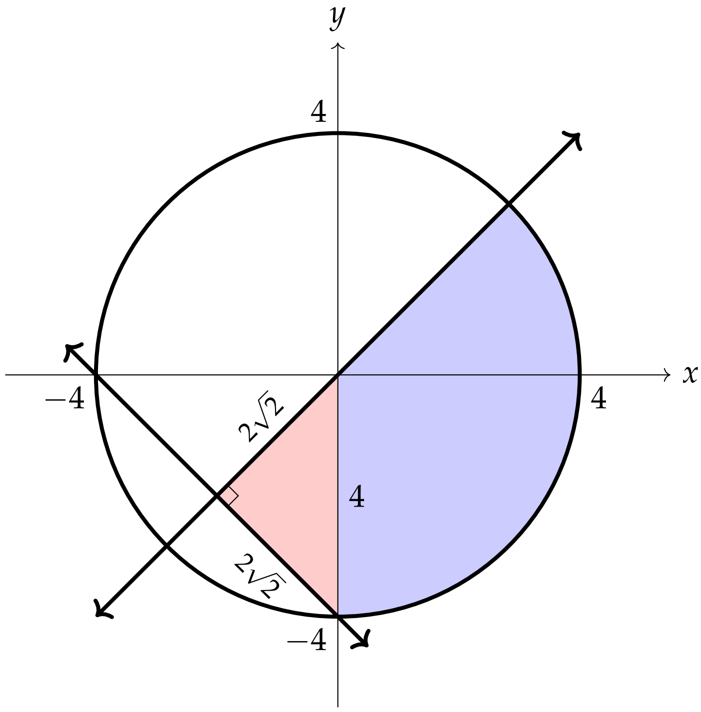
18. In the xy-plane, the graphs of the equations $y = x$ and $y = -x - 4$ divide the circle whose equation is $x^2 + y^2 = 16$ into four regions, consisting of

two congruent, larger regions and two congruent, smaller regions. The area of one of the larger regions is $m\pi + n$ square units, for some integers m and n. What is $m + n$?

Answer (10): We start by graphing all three equations. The graph of $x^2 + y^2 = 16$ is a circle centered at the origin whose radius is $\sqrt{16} = 4$. The desired region is shown below:

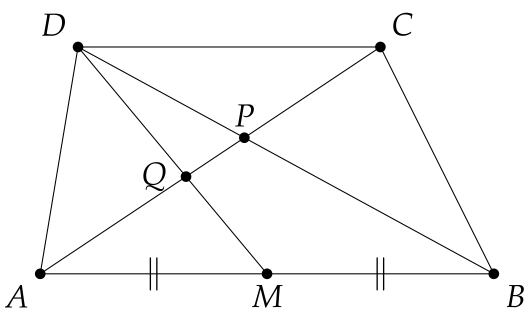


We can split the above shaded region along the y-axis, forming a 135° sector of a circle whose radius is 4, and an isosceles right triangle whose hypotenuse has length 4:



The area of the 135° sector is $\frac{135}{360} \times 4^2\pi = 6\pi$ square units. The isosceles triangle has leg lengths $2\sqrt{2}$, and its area is $\frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 4$ square units. The area of the entire shaded region is $6\pi + 4$ square units, so $m + n = 6 + 4 = 10$.

19. In trapezoid $ABCD$ with parallel bases $\overline{AB}$ and $\overline{CD}$, point M is the midpoint of $\overline{AB}$, point P is the intersection of diagonals $\overline{AC}$ and $\overline{BD}$, and point Q is the intersection of diagonal $\overline{AC}$ and segment $\overline{DM}$. Given that $AB = 6$ and $CD = 4$, what integer is closest to the ratio of the area of trapezoid $ABCD$ to the area of $\triangle DPQ$?



Answer (15): We will first find the ratio of the area of $\triangle ACD$ to the area of $\triangle DPQ$. Both triangles share a common altitude (the altitude from D onto $\overline{AC}$), so this ratio also equals the ratio $AC : PQ$.

By AA similarity, $\triangle ABP \sim \triangle CDP$ with a $6 : 4$ scale factor, and $\triangle AMQ \sim \triangle CDQ$ with a $3 : 4$ scale factor. This implies $AP : CP = 3 : 2$, and $AQ : CQ = 3 : 4$. We find that $AQ = 15k$, $QP = 6k$, and $CP = 14k$ for some k. Therefore $AC : PQ = 35 : 6$, so the ratio of the area of $\triangle ACD$ to the area of $\triangle DPQ$ is $\frac{35}{6}$.

Next, we find the ratio of the area of trapezoid $ABCD$ to the area of $\triangle ACD$. Suppose the height of $ABCD$ is h. Using the formula for the area of a trapezoid, the area of $ABCD$ is $\frac{1}{2}h(6+4) = 5h$, while the area of $\triangle ACD$ is $\frac{1}{2}(4h) = 2h$. The ratio of the area of $ABCD$ to the area of $\triangle ACD$ is $\frac{5}{2}$.

Finally, the desired ratio equals

$$\begin{aligned}\frac{\text{area of } ABCD}{\text{area of } \triangle DPQ} &= \frac{\text{area of } ABCD}{\text{area of } \triangle ACD} \times \frac{\text{area of } \triangle ACD}{\text{area of } \triangle DPQ} \\ &= \frac{5}{2} \times \frac{35}{6} \\ &= \frac{175}{12} = 14\frac{7}{12}.\end{aligned}$$

The integer closest to this ratio is 15.

20. A digital scale displays weights in grams (g), with a random error of ≤ 5 g from the actual weight. Jo has one apricot, one lime, and one plum, each weighing a whole number of grams. When she places the apricot and lime on the scale, the scale displays 87 g. When she places the apricot and plum on the scale, the scale displays 85 g. Finally, when she places the lime and plum on the scale, the scale displays 106 g. What is the smallest possible weight (in grams) of the apricot?

Answer (26): Let a, ℓ, and p denote the actual weights of the apricot, lime, and plum, respectively (in grams). From the given information, we can form the following inequalities:

$$\begin{aligned}82 \leq a + \ell \leq 92 \\ 80 \leq a + p \leq 90 \\ 101 \leq \ell + p \leq 111\end{aligned}$$

Ordinarily, if we were given the exact values of $a+\ell$, $a+p$, and $\ell+p$, we can easily solve for a by adding the values of $a+\ell$ and $a+p$ and subtracting the value of $\ell+p$. Instead, in order to make a as small as possible, we will minimize $a+\ell$ and $a+p$, and maximize $\ell+p$.

Consider the lower bounds for $a+\ell$, $a+p$, as well as the upper bound for $\ell+p$. We will multiply both sides of the inequality $\ell+p \leq 111$ by -1 to obtain the equivalent inequality $-\ell-p \geq -111$:

$$\begin{aligned} a+\ell &\geq 82 \\ a+p &\geq 80 \\ -\ell-p &\geq -111 \end{aligned}$$

Adding all three inequalities and dividing by 2 shows that $a \geq 25.5$. Since each fruit weighs a whole number of grams, the smallest possible weight of the apricot is $a=26$. One possible solution is $(a,\ell,p)=(26,57,54)$.

21. How many positive integers are factors of both $10^{2022}-1$ and $10^{2025}-1$?

Answer (08): We can look at the greatest common divisor of $10^{2022}-1$ and $10^{2025}-1$, since any positive integer divisor of both numbers must be a divisor of their GCD. Using the fact that $\gcd(a,b)=\gcd(a,b-a)$, we can compute $\gcd(10^{2022}-1,10^{2025}-1)$ rather quickly:

$$\begin{aligned} \gcd(10^{2022}-1,10^{2025}-1) &= \gcd(10^{2022}-1,(10^{2025}-1)-(10^{2022}-1)) \\ &= \gcd(10^{2022}-1,10^{2025}-10^{2022}) \\ &= \gcd(10^{2022}-1,10^{2022}(10^3-1)). \end{aligned}$$

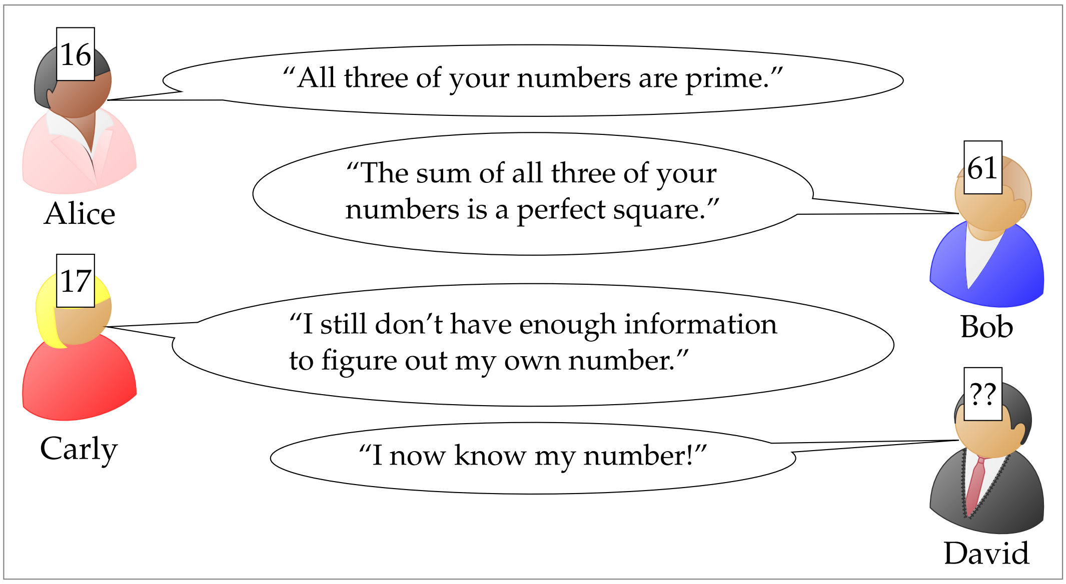
We observe that $10^{2022}-1$ and 10^{2022} share no prime factors in common since these integers differ by 1. Therefore, the desired GCD equals $\gcd(10^{2022}-1,10^3-1)=\gcd(10^{2022}-1,999)$. Note that the number $10^{2022}-1$ consists of 2022 9's. Moreover, every number of the form 999, 999999, 999999999, ..., where the number of 9's is a multiple of 3, is divisible by 999. As 2022 is divisible by 3, we can conclude that $10^{2022}-1$ is divisible by 999, so therefore the desired GCD equals 999; that is,

$$\gcd(10^{2022}-1,10^{2025}-1)=999.$$

The positive integers which are divisors of both numbers are necessarily the divisors of 999. The prime factorization of 999 is $3^3\times 37^1$, so 999 has

$(3+1)(1+1) = 8$ factors and we conclude that the answer is 8.

22. A deck contains 100 cards numbered 1 through 100. Alice, Bob, Carly, and David are each given a different card. Each person tapes their card to their forehead, so that they can see everyone's cards except their own. They have the following conversation:



Assuming everyone speaks truthfully, what number is on David's card?

Answer (31): Let d be the number on David's card. From Alice's and Bob's statements, David will know that d is prime, and that $16+17+d = 33+d$ is a perfect square. Therefore, David knows that his number is either 3, 31, or 67. We will show that if $d = 3$ or 67, Carly would have been able to figure out her own number, contradicting her statement.

Suppose $d = 3$, and let c be the number on Carly's card (we know that $c = 17$). Carly, seeing Alice's and David's cards, would know from Bob's statement that $16+3+c = 19+c$ is a perfect square, and would also know from Alice's statement that c is prime. The only integer c (with $1 \leq c \leq 100$) satisfying both of these conditions is $c = 17$. Therefore, if $d = 3$, Carly would have been able to determine that $c = 17$.

Suppose $d = 67$. Similarly, Carly would know from Bob's statement that $16+67+c = 83+c$ is a perfect square, and deduce that $c = 17$ or 61.

However, Carly also sees that Bob already has the number 61, so Carly would also have been able to determine that $c = 17$.

Lastly, suppose $d = 31$. Similarly, Carly would know from Bob's statement that $16 + 31 + c = 47 + c$ is a perfect square, but would only be able to deduce that $c = 17, 53$, or 97. In this case, Carly would not have enough information to figure out her number. Therefore, David can deduce that he has the number 31.

23. The 4-digit integer 2025 has the property that, when split between the hundreds and tens digits to form two 2-digit integers (20 and 25), the second integer is 5 greater than the first integer. Other than 45, there is exactly one other positive 2-digit integer n such that n^2 is a 4-digit integer with this property. What is n?

Answer (56): In order for n^2 to satisfy the given property, we require $n^2 = \overline{abcd}$, where $\overline{cd} = \overline{ab} + 5$. Let $\overline{ab} = x$, so that $\overline{cd} = x + 5$. Then $n^2 = 100x + (x + 5) = 101x + 5$; using modular arithmetic, this can be written as $n^2 \equiv 5 \pmod{101}$.

We are given that $n = 45$ satisfies the given property, so $n = 45$ satisfies the congruence $n^2 \equiv 5 \pmod{101}$. We observe that n is a solution to the congruence $n^2 \equiv 5 \pmod{101}$ if and only if $101 - n$ is a solution. This follows as $(101 - n)^2 = 101^2 - 2(101)n + n^2 \equiv n^2 \pmod{101}$. Therefore $n = 101 - 45 = 56$ is also a solution with $56^2 = 3136$; moreover, $n = 56$ is the only solution.

24. When the fraction $\dfrac{a}{b}$ is rounded to the nearest hundredth, the result is 0.51. Given that a and b are 2-digit positive integers, what is the smallest possible value of $a + b$?

Answer (53): Noting that 0.51 is approximately $\frac{1}{2}$, we want to find integers a and b such that $b \approx 2a$. Specifically, b must be slightly less than $2a$, since $\frac{a}{b}$ is greater than $\frac{1}{2}$, and the fraction $\frac{a}{b}$ must be between $0.505 = \frac{101}{200}$ and $0.515 = \frac{103}{200}$. We remark that 0.505 by convention rounds up to 0.51, but this does not produce a valid solution (a, b).

To minimize b, we claim b should be odd (see Remark below). Letting $b = 2k - 1$ for some integer $k \geq 1$, we set $a = k$ so that $\frac{a}{b}$ is slightly larger than $\frac{1}{2}$. We can then solve the inequality $\frac{101}{200} < \frac{k}{2k-1} < \frac{103}{200}$ for k:

$$\frac{101}{200} < \frac{k}{2k-1} < \frac{103}{200}$$

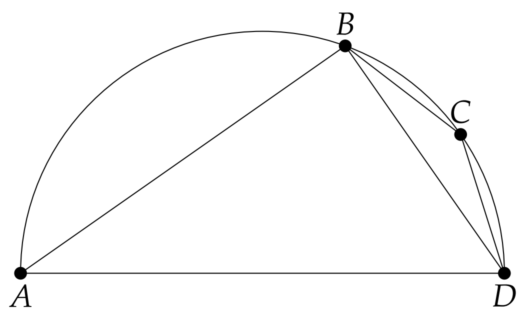
Multiply both sides by $200(2k-1)$:

$$101(2k-1) < 200k < 103(2k-1)$$
$$202k - 101 < 200k < 206k - 103$$

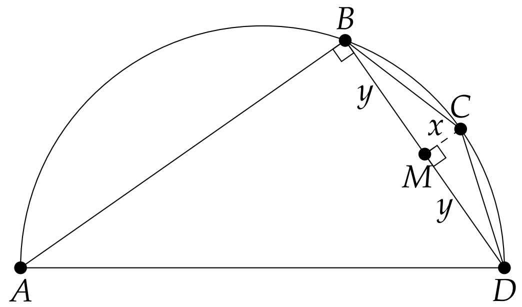
Solving both inequalities for k yields $17.1\overline{6} < k < 50.5$. In order to minimize b, we must minimize k; by setting $k = 18$, we obtain $b = 2k - 1 = 35$. The corresponding fraction is $\frac{18}{35} \approx 0.5143$, and $a + b = 18 + 35 = 53$.

Remark: If b is even, then $b = 2k$ for some integer k. In order to minimize b, we must set $a = k + 1$ so that $\frac{a}{b}$ is slightly larger than $\frac{1}{2}$. Solving the inequality $\frac{101}{200} < \frac{k+1}{2k} < \frac{103}{200}$ for k in a similar way yields $33.\overline{3} < k < 100$, so the minimum integer k which satisfies this inequality is $k = 34$, corresponding to $b = 68$. As $68 > 35$, setting b to be even does not yield the minimum solution.

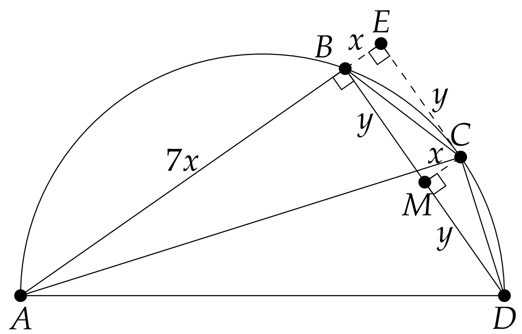
25. Points B and C are on a semicircle with diameter $\overline{AD}$, so that B is closer than C to A, and $BC = CD$. Given that the areas of $\triangle ABD$ and $\triangle BCD$ are $56\sqrt{2}$ cm^2 and $8\sqrt{2}$ cm^2, respectively, what is the perimeter of quadrilateral $ABCD$, in cm?



Answer (44): We first observe that $m\angle ABD = m\angle ACD = 90^\circ$ since $\overline{AD}$ is a diameter. Let M be the midpoint of $\overline{BD}$. Since $\triangle BCD$ is isosceles, it follows that $\overline{CM} \perp \overline{BD}$. Let $CM = x$ and $BM = DM = y$:



The area of $\triangle BCD$ is xy, so $xy = 8\sqrt{2}$. Next, since $\triangle ABD$ has area equal to seven times that of $\triangle BCD$, and $\overline{AB}$ is an altitude of $\triangle ABD$, it follows that $AB = 7x$. To find x and y, we will find the length of $\overline{AC}$ two different ways. First, extend $\overline{AB}$ past B to a point E such that $BMCE$ is a rectangle:



By the Pythagorean theorem on $\triangle AEC$, we have $AC^2 = (8x)^2 + y^2 = 64x^2 + y^2$. We can also find AC^2 using the Pythagorean theorem on $\triangle ABD$ and $\triangle ACD$: noting that $AD^2 = AB^2 + BD^2 = 49x^2 + 4y^2$ and $CD^2 = x^2 + y^2$, we have

$$\begin{aligned} AC^2 &= AD^2 - CD^2 \\ &= (49x^2 + 4y^2) - (x^2 + y^2) \\ &= 48x^2 + 3y^2. \end{aligned}$$

We now have two different expressions for AC^2; equating them gives $64x^2 + y^2 = 48x^2 + 3y^2$, which simplifies to $2y^2 = 16x^2$ or $y = 2\sqrt{2}x$.

Using $xy = 8\sqrt{2}$, we have $x(2\sqrt{2}x) = 8\sqrt{2} \implies x = 2$. Then $y = 2\sqrt{2}x = 4\sqrt{2}$. We now compute the lengths of $ABCD$, namely $BC = CD = \sqrt{x^2 + y^2} = 6$, $AB = 7x = 14$, and $AD = \sqrt{AB^2 + BD^2} =$

$\sqrt{14^2 + (8\sqrt{2})^2} = 18$. The perimeter of quadrilateral $ABCD$ is $6 + 6 + 14 + 18 = 44$.