Pi Math Contest Euler Division

2025 Solutions

Solutions

1. What is the value of $\dfrac{1}{\frac{2}{3}-\frac{1}{2}}$?

   Answer (6): To evaluate the expression $\frac{1}{\frac{2}{3}-\frac{1}{2}}$, we first find a common denominator for the fractions in the denominator:

   $$\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}=\frac{1}{6}.$$

   To divide by a fraction, we can multiply by its reciprocal:

   $$\frac{1}{\frac{2}{3}-\frac{1}{2}}=\frac{1}{\frac{1}{6}}=1\times\frac{6}{1}=6.$$

   The desired answer is 6.

2. Apples are regularly priced at \$1 each, but are 30% off today. How many dollars will Amelia spend for ten apples, with the discount applied?

   Answer (7): Calculate the discounted price of one apple:

   $$\begin{aligned}\text{Discounted price} &= \$1-(30\%\times\$1)\\ &= \$1-\$0.30\\ &= \$0.70.\end{aligned}$$

   Multiply the discounted price by the number of apples:

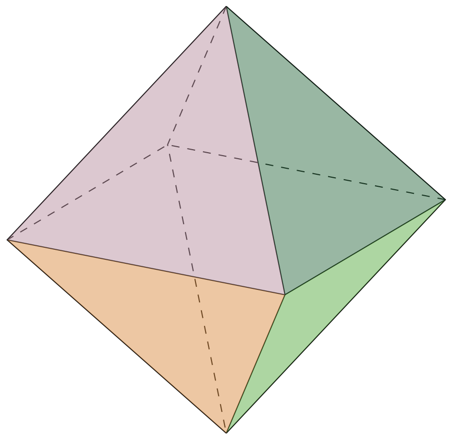
   $$\begin{aligned}\text{Total cost} &= \$0.70\times 10\\ &= \$7.00.\end{aligned}$$

Amelia will spend \$7 for 10 apples.

3. Jamie has three shirts and three pairs of pants. How many ways can he choose one shirt and one pair of pants to wear today?

   Answer (9): Jamie has 3 choices for a shirt and 3 choices for a pair of pants. By the multiplication principle, he has $3 \times 3 = 9$ ways to choose one shirt and one pair of pants.

4. In the 3-dimensional shape below, what is the result when the number of faces and the number of vertices are added, and the number of edges is subtracted from this sum?



   Answer (2): The given shape (called an *octahedron*) has 8 faces, 6 vertices, and 12 edges. The desired answer is $8 + 6 - 12 = 2$.

   Remark: For any convex polyhedron with F faces, V vertices, and E edges, the equation $F + V - E = 2$ is true.

5. Samantha bakes cookies to raise funds for a charity event. She can bake 12 cookies with 1 cup of flour. She plans to bake 108 cookies today. If Samantha already has 3 cups of flour, how many more cups of flour does she need in order to bake all 108 cookies?

   Answer (6): If she can bake 12 cookies with 1 cup of flour, then the number of cups of flour Samantha needs is

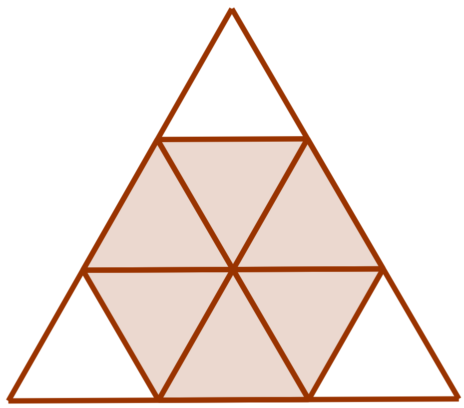
$$\frac{108 \text{ cookies}}{12 \text{ cookies/cup}} = 9 \text{ cups.}$$

Since Samantha already has 3 cups of flour, she needs $9 - 3 = 6$ additional cups of flour to bake all 108 cookies.

6. Jessica is tidying up her toy collection. She places $\frac{3}{4}$ of her toys into a colorful toy chest. She then uses $\frac{2}{5}$ of the remaining toys to create a play scene, and places the remaining toys in a basket. If Jessica has 60 toys altogether, how many toys are placed in the basket?

   Answer (9): Jessica places $\frac{3}{4} \times 60$, or 45 toys in her toy chest. Therefore she has $60 - 45 = 15$ toys remaining. Of these 15 toys, she uses $\frac{2}{5} \times 15 = 6$ toys to create a play scene, and places $15 - 6 = 9$ toys in the basket.

7. The triangle shown below has area 12, and is divided into smaller, identical triangles. What is the area of the shaded region?

   Answer (8): The large triangle consists of 9 smaller triangles, of which 6 are shaded. The area of the shaded region is $\frac{6}{9} \times 12 = 8$.

8. A jar filled with peanut butter weighs 12 pounds. When one third of the peanut butter is removed, the jar and the remaining peanut butter weigh $9\frac{2}{3}$ pounds. How many pounds does the empty jar weigh?

   Answer (5): First, we will calculate the weight of the peanut butter that was removed:

$$\begin{aligned} 12 - 9\frac{2}{3} &= 12 - \frac{29}{3} \\ &= \frac{36}{3} - \frac{29}{3} \\ &= \frac{7}{3} \text{ lb.} \end{aligned}$$

This quantity represents one third of the total weight of the peanut butter. To find the weight of the peanut butter, we multiply by 3:

$$\text{weight of peanut butter} = \frac{7}{3} \times 3 = 7 \text{ lb.}$$

The weight of the empty jar is $12 - 7 = 5$ pounds.

9. Given the 10-digit number 1024687935, Sam erases a digit to create the smallest 9-digit number possible. Given that this 9-digit number cannot begin with "0," what digit does Sam remove?

   Answer (8): Since Sam cannot create a 9-digit number starting with "0," she cannot erase the digit "1." We can examine the resulting numbers when Sam erases any other digit:

$$\begin{aligned} &\text{erase } 0 \to 124687935 \\ &\text{erase } 2 \to 104687935 \\ &\text{erase } 4 \to 102687935 \\ &\text{erase } 6 \to 102487935 \\ &\text{erase } 8 \to 102467935 \end{aligned}$$

   Erasing the digit 8 yields a number beginning with 102467, which is minimum. Erasing any of the odd digits will result in a number beginning with 102468, which is larger. We conclude that Sam removes the digit "8."

10. Thirty-eight students went on a field trip. On the way there, 20 students traveled in a large bus, while the remaining students traveled in cars, with 3 students per car. For the return trip, the same vehicles were used, and all students returned, but each car carried 5 students instead. How many students rode the bus on the way back?

    Answer (8): On the way to the field trip, 20 students traveled in the bus, so

$38 - 20 = 18$ students rode in cars. Since there were 3 students per car, the number of cars is $18 \div 3 = 6$.

On the way back, since 6 cars were used and 5 students rode in each car, there are $6 \times 5 = 30$ students who rode in cars. Since all 38 students returned, the number of students who returned on the bus is $38 - 30 = 8$.

11. Omar has an appointment scheduled for 2:00 PM, which is located 48 miles away from his house. Omar departs his house at 12:30 PM and maintains an average speed of 30 miles per hour. How many minutes late will he be for his appointment?

Answer (6): We will calculate the time it takes for Omar to travel to his appointment, using $d = rt$:

$$\text{time} = \frac{48 \text{ miles}}{30 \text{ mi/hr}} = \frac{8}{5} \text{ hr} = 96 \text{ minutes.}$$

Therefore, Omar will arrive at his appointment 96 minutes, or 1 hour 36 minutes, after 12:30 PM, which is 2:06 PM. Since his appointment is scheduled for 2:00 PM, he will be 6 minutes late.

12. The table below gives the price per pound (lb) for four different types of nuts:

Nuts	**Price/lb**
Pistachio	\$9
Cashews	\$5
Almond	\$3
Peanut	\$1

Alice creates a mix using 7 lb of pistachios, 2 lb of cashews, 6 lb of almonds, and 4 lb of peanuts. In dollars, what is the price per pound of Alice's mix?

Answer (5): Let's calculate the total cost of each type of nut:

$$\text{Pistachio: } 7 \text{ lb} \times \$9/\text{lb} = \$63$$
$$\text{Cashews: } 2 \text{ lb} \times \$5/\text{lb} = \$10$$
$$\text{Almond: } 6 \text{ lb} \times \$3/\text{lb} = \$18$$
$$\text{Peanut: } 4 \text{ lb} \times \$1/\text{lb} = \$4$$

The total cost of Alice's mix is $\$63 + \$10 + \$18 + \$4 = \$95$. The total weight of Alice's mix is $7 + 2 + 6 + 4 = 19$ lb. Therefore, the price of Alice's mix (per pound) is

$$\text{Price of mix per pound} = \frac{\text{Total cost of mixture}}{\text{Total weight of mixture}} = \frac{\$95}{19\text{ lb}} = \$5/\text{lb}.$$

Alice's mix costs $5 per pound.

13. A total of 525 tomato plants were planted in a garden over a period of 21 days. If an equal number of tomato plants were planted each day for the first 20 days, what is the fewest number of tomato plants that could have been planted on the 21^{st} day?

Answer (5): To minimize the number of plants that were planted on the 21^{st} day, we must maximize the number of plants that were planted on each of the first 20 days. To do so, we divide $525 \div 20$ and only keep the quotient:

$$525 \div 20 = 26 \text{ remainder } 5.$$

Therefore, if 26 tomato plants were planted on each of the first 20 days, then 5 plants will be planted on the 21^{st} day, which is the minimum.

14. The 5-digit number $53,00A$ is divisible by 11. What is the value of the digit A?

Answer (9): Dividing $53000 \div 11$ yields a quotient of 4818 with a remainder of 2. Therefore, by adding 9 to 53000, we obtain a multiple of 11. The digit A must equal 9.

Alternate solution: In general, a positive integer is divisible by 11 if and only if the difference between the sum of odd-positioned digits and the sum of the even-positioned digits is a multiple of 11. For the number $53,00A$, this difference is $(5 + 0 + A) - (3 + 0) = A + 2$. Therefore, $A + 2$ must be divisible by 11. Since A is a digit between 0 and 9, the only possible value for the digit A is 9.

15. In how many ways can 5 be written as a sum of one or more positive integers, where the order of the integers does not matter? For example, $3 + 1 + 1$ is the same as $1 + 3 + 1$.

Answer (7): There are 7 ways to write 5 as a sum of one or more positive integers where the order of the integers does not matter. They are:

- 5
- 4+1
- 3+2

- 3+1+1
- 2+2+1
- 2+1+1+1
- 1+1+1+1+1.

16. A farmer is constructing a rectangular horse pen with dimensions 12 feet by 18 feet. He plans to place a fence post at every corner of the pen, and additional fence posts every 3 feet along each side. If the farmer has 22 fence posts, how many fence posts will be remaining once he finishes the pen?

    Answer (2): Since 12 and 18 are evenly divisible by 3, a fence post will be placed every 3 feet along the perimeter of the pen, including at the corners of the pen.

    The perimeter of the horse pen is $2 \times (12 + 18) = 60$ feet, so the number of posts required is $60 \div 3 = 20$. Therefore, 20 fence posts are needed to complete the job. Since the farmer has 22 fence posts to start with, he will have $22 - 20 = 2$ fence posts once he finishes.

17. In the addition below, each letter represents a different nonzero digit:

$$\begin{array}{rcccc} & A & A & A & A \\ & & B & B & B \\ & & & C & C \\ + & & & & D \\ \hline & 2 & 0 & 2 & 5 \end{array}$$

    What is the value of D?

    Answer (4): First, the value for digit A must be 1. If A was equal to 2, then the sum would be greater than 2222 and cannot be equal to 2025.

    Next, we claim that $B = 8$. If $B = 9$, then $1111 + 999 = 2110 > 2025$. On the other hand, if $B \leq 7$, then $1111 + BBB \leq 1111 + 777 = 1888$. However, it is impossible to add a 2-digit number and a 1-digit number to 1888 to achieve a sum of 2025, since $1888 + 99 + 9 < 2025$. Therefore, $B = 8$, and the sum of the first two numbers is $1111 + 888 = 1999$.

    The remaining two numbers, CC and D, must add to $2025 - 1999 = 26$. Therefore, $C = 2$ and $D = 4$, and the solution is $1111 + 888 + 22 + 4 = 2025$.

18. A teacher wishes to arrange 18 drama students and 24 choir students into equal-sized rows. Every row will contain the same number of drama students, and every row will contain the same number of choir students. Note that there will be more choir students than drama students in each row. Given that the teacher maximizes the number of rows, how many students will be in each row?

    Answer (7): The number of rows must be a factor of 18 and 24, since the 18 drama students are evenly divided among the rows, and the 24 choir students are also evenly divided among the rows. The greatest common divisor of 18 and 24 is
    $$\gcd(18, 24) = \gcd(2^1 \times 3^2, 2^3 \times 3^1) = 2^1 \times 3^1 = 6.$$
    Therefore, the teacher can form at most 6 rows, which is also the maximum number of rows. Each row will therefore have $18 \div 6 = 3$ drama students and $24 \div 6 = 4$ choir students, and each row will have a total of $3 + 4 = 7$ students.

19. The length and width of a rectangle each measure a positive whole number of inches, and the area of the rectangle is 135 square inches. How many different values are possible for the perimeter of the rectangle?

    Answer (4): To find the possible perimeters of a rectangle with an area of 135 square inches, where each side length is a positive whole number of inches, we find all factor pairs which multiply to 135:
    $$135 = 1 \times 135 = 3 \times 45 = 5 \times 27 = 9 \times 15.$$
    Each of these four factor pairs will produce a rectangle with a different perimeter (272, 96, 64, and 48 inches, respectively). We conclude that there are 4 values for the perimeter of the rectangle.

20. A classroom has 23 girls and 12 boys. The teacher divides the students into groups, ensuring that each group has no more than 3 more girls than boys. What is the minimum number of groups needed?

    Answer (4): The difference between the number of girls and the number of boys is $23 - 12 = 11$. Since each group can have at most 3 more girls than boys, we need to distribute these 11 extra girls among the groups. Dividing $11 \div 3$ gives 3 extra girls per group with 2 extra girls in a fourth group. Therefore, the teacher must create at least 4 groups. One possible way is for three of the groups to have 6 girls and 3 boys, and the fourth group to have 5 girls and 3 boys.

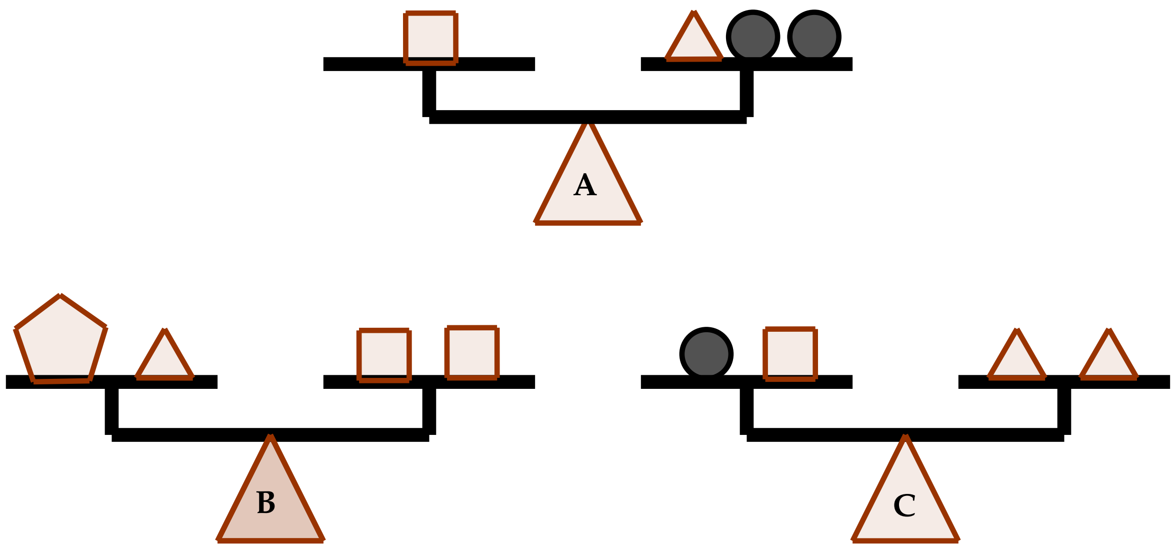
21. Twenty-four students participated in a math contest. Adam placed 19^{th}, and Mary outperformed three times as many students as Adam did. Given that no

ties occurred, how many students performed better than Mary?

Answer (8): Since Adam placed 19^{th}, he performed better than 5 students (the students who placed 20^{th} through 24^{th}). Since Mary outperformed three times as many students as Adam, it follows that Mary performed better than $3 \times 5 = 15$ students.

Therefore, Mary must have performed better than the students in 10^{th}, 11^{th}, $\ldots$, 24^{th} place. Then Mary placed 9^{th}, and 8 students performed better than Mary.

22. Three balance scales (A, B, and C) are given below:



How many circles are needed to balance one pentagon?

Answer (7): We will use the fact that one square balances one triangle and two circles (from scale A). Suppose we replace the square in scale C with a triangle and two circles. It follows that three circles and one triangle balance two triangles; removing a triangle from both sides implies that three circles balance one triangle.

Going back to scale A, we can replace the triangle with three circles to show that one square balances five circles.

Going to scale B, we can replace one triangle with 3 circles and two squares with $2 \times 5 = 10$ circles. Therefore, a pentagon and 3 circles will balance 10 circles. Removing 3 circles on each side, we conclude that a pentagon will balance 7 circles.

23. Eleven coins lie on a table, all showing heads. In each move, you select any three coins (not necessarily consecutive) and flip them. What is the minimum number of moves needed to turn all 11 coins to tails?

Answer (5): At least 4 moves are required, since 3 moves will only change at most 9 of the coins from heads to tails. We claim that 4 moves is impossible. To show this, note that in order to make any given coin change from heads, it must be flipped an odd number of times (e.g., 1 or 3 times). Since there are 11 coins, the total number of times that some coin is flipped must be equal to the sum of 11 odd numbers, which is odd. But with 4 moves, the number of times that some coin is flipped is $4 \times 3 = 12$, which is even. This shows that it is impossible to do this task in exactly 4 moves.

The following table shows that 5 moves is possible, therefore showing that the answer is 5.

	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
Initial	H	H	H	H	H	H	H	H	H	H	H
Move 1	**T**	**T**	**T**	H	H	H	H	H	H	H	H
Move 2	T	T	T	**T**	**T**	**T**	H	H	H	H	H
Move 3	T	T	T	T	T	T	**T**	**T**	**T**	H	H
Move 4	T	T	T	T	T	T	T	**H**	**H**	**T**	H
Move 5	T	T	T	T	T	T	T	**T**	**T**	T	**T**

24. A valid credit card number consists of 16 digits 0-9, where the leftmost digit is the 1^{st} digit. The first 15 digits are considered random, while the 16^{th} digit is a "check digit," calculated using the following steps:

    I. Add all the digits in the odd-numbered positions.

    II. Double the result from step I.

    III. Add all the digits in the even-numbered positions (excluding the check digit).

    IV. Count the number of odd-positioned digits that are greater than 4.

    V. Sum the resulting numbers obtained from steps II, III, and IV.

    VI. Determine the digit needed to add to the result from step V to reach the next multiple of 10. This digit is the check digit.

    What digit should replace the blank in the credit card number below so that the resulting number is valid?

    4783 1980 2569 290_

Answer (3): Given the first 15 digits: 4783 1980 2569 290_:

I. **Odd-numbered positions:** 4, 8, 1, 8, 2, 6, 2, 0
Sum of digits: $4 + 8 + 1 + 8 + 2 + 6 + 2 + 0 = \mathbf{31}$

II. **Double the result from step I:** $31 \times 2 = \mathbf{62}$

III. **Even-numbered positions:** 7, 3, 9, 0, 5, 9, 9
Sum of even-positioned digits: $7 + 3 + 9 + 0 + 5 + 9 + 9 = \mathbf{42}$

IV. **Number of odd-positioned digits greater than 4: 3** (8, 8, and 6)

V. **Sum of numbers from steps II, III, and IV:** $62 + 42 + 3 =$ **107**

VI. **Determine the check digit:** The next multiple of 10 after 107 is 110. The check digit is $110 - 107 =$ **3**.

25. Asha has 20 indistinguishable coins. 19 of them have the same weight, while the remaining coin is slightly heavier. Using a two-pan balance scale, what is the fewest number of measurements Asha needs to do in order to guarantee that she will identify the heavier coin? (A *measurement* consists of placing some coins on both ends of the scale, and recording the heavier or that both sides have the same weight.)

    Answer (3): To identify the heavier coin among 20 indistinguishable coins, a minimum of 3 measurements is necessary. The following algorithm shows one way to do so. The main idea behind this algorithm is to repeatedly divide the coins into three equal-sized or nearly equal-sized groups so that the number of coins to consider decreases by a factor of 3 with each measurement.

    a) Divide the coins into three groups: two with 7 coins each, and one with 6 coins.

    b) Measure the two groups with 7 coins each.
       - If the scale is balanced, the heavier coin must be in the group with 6 coins.
       - Otherwise, the heavier coin must be in the heavier of the two groups of 7 coins.

    c) Take the group of 6 or 7 coins identified from step b) which contains the heavier coin.

    d) If this group has 6 coins:
       - Divide the coins into three groups of 2 coins each. Weigh two of the three groups.
       - If the scale is balanced, the heavier coin is in the third group. Weigh these two coins to determine the heavier coin.
       - Otherwise, the heavier coin is in one of the first two groups. Weigh the two coins in this group to determine the heavier coin.

    e) If this group has 7 coins:
       - Divide the coins into groups of 2, 2, and 3. Weigh the groups containing 2 coins each.

- If the scale is balanced, the heavier coin is in the third group containing 3 coins. Weigh two of these 3 coins and take the heavier coin (or the third coin if the weights are equal).
- Otherwise, the heavier coin is in one of the first two groups; weigh the 2 coins to determine the heavier coin.

Therefore, the heavier coin can be found with 3 measurements.

Remark: To show that two weighings is impossible, note that each weighing can result in three possible outcomes: the left side is heavier, the right side is heavier, or both sides are balanced.

With 2 weighings, we can distinguish at most $3^2 = 9$ outcomes. Since there are 20 coins, which is greater than 9, it is impossible to identify the heavier coin in just two weighings.