Pi Math Contest - Gauss Division Sample Problems

1.

$$\sqrt{9 \cdot 10 \cdot 11 \cdot 12 + 1} = ?$$

Answer: | 109

Solution:

$$\sqrt{9 \cdot 10 \cdot 11 \cdot 12 + 1} = \sqrt{108 \cdot 110 + 1}$$

$$= \sqrt{(109 - 1)(109 + 1) + 1}$$

$$= \sqrt{109^2 - 1^2 + 1}$$

$$= \sqrt{109^2 - 1 + 1}$$

$$= \sqrt{109^2}$$

$$= 109$$

2. What is the greatest common divisor of 246, 123 and 2, 460, 123?

Answer: | 369

Solution: solution

3. Maya is using an escalator that is moving up. If she goes up the escalator at a constant speed, it takes her 60 seconds. If she goes down the escalator at the same constant speed, it takes her 120 seconds. How many seconds will it take her to go up the escalator if she stays still on the escalator?

Answer: 240

Solution: Let e and v be the speeds of the escalator and Maya, respectively.

Calculating the length of the escalator we have

$$60(e+v) = 120(v-e) = t \cdot e$$
,

where t is the number of seconds it takes to go up the moving escalator when she is still.

From the first equation, we get

$$e + v = 2v - 2e \implies v = 3e$$
.

Plugging it above, the equations turn into

$$60 \cdot 4e = t \cdot e \implies t = 240.$$

4. Alex has 5 different donuts. Each day he eats at least one donut. In how many different ways can he finish his donuts?

Answer: 541

Solution: solution

5. Triangle $\triangle ABC$ has side lengths $\overline{AB}=15$, $\overline{BC}=20$ and $\overline{AC}=25$. A semicircle is drawn passing through the points A,B and C. Two more semicircles, diameters \overline{AB} and \overline{BC} are drawn from outside the right triangle. Find the total area that is inside the second or third semicircle but outside the first semicircle.

Answer: 150

Solution: solution