Pi Math Contest - Fermat Division
Sample Problems

1. Find the remainder when 20182018 is divided by 101.

   Answer: $\boxed{98}$

   Solution: Through long division, we get $20182019 = 101 \times 199821 + 98$. Hence, the answer is 98. Alternatively, note that 100 is 1 less than 101. This leads to even powers of 100 being 1 more than and odd powers of 100 being 1 less than a multiple of 101. Writing 20182019 in base 100, we get:

   $$20 \times 100^3 + 18 \times 100^2 + 20 \times 100 + 19.$$

   This number differs from

   $$19 - 20 + 18 - 20 = -3.$$

   by a multiple of 101. Hence, it differs from $101 - 3 = 98$ by a multiple of 101 as well. Which means the remainder is 98.

2. In triangle $\triangle ABC$, one of the angles is twice another. If $\angle A = 54°$, what is the sum of all possible values of the smallest angle of this triangle, in degrees?

   Answer: $\boxed{87}$

   Solution: Depending on which angle is $\angle A$ (half another angle, twice another angle, or the third angle) we have the following triples for the angles of this triangle:

   $$(54°, 108°, 18°)$$

   $$(27°, 54°, 99°)$$

   $$(42°, 84°, 54°).$$

   So, the smallest angle can be $18°$, $27°$ or $42°$. The answer is $18 + 27 + 42 = 87$.

3. The roots of the polynomial $x^3 - 12x^2 + ax - 28$ form an arithmetic sequence. What is a?

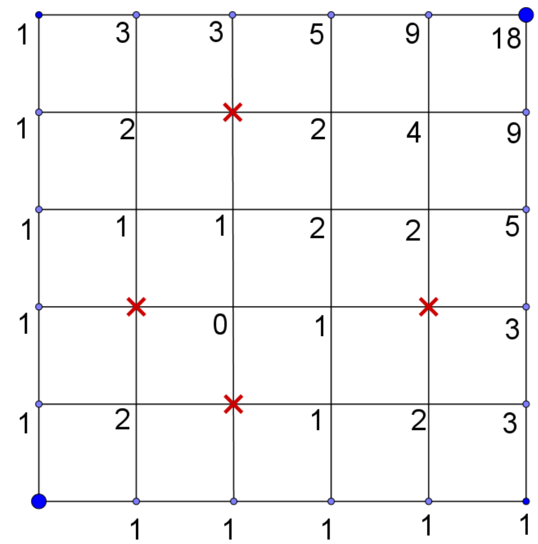
   Answer: $\boxed{39}$

   Solution: Let the roots be $r - d, r$ and $r + d$. By Vieta, the sum of the roots is $3r = 12$. So, $r = 4$ is one of the roots. Thus, $4^3 - 12 \cdot 4^2 + a \cdot 4 - 28 = 0$ and $a = 39$.

4. Alpha the Penguin will take a shortest path in a grid from $(0, 0)$ to $(5, 5)$. Along her path, except at the starting point, she never goes through a point one of whose coordinates is twice the other. How many different shortest paths can she take with this condition?

   Answer: $\boxed{18}$

Solution: Alpha is walking from lower left end to upper right end in the following grid. The points that Alpha is avoiding are marked with an X. Starting with the points she can get in one step, on each corner we write down the number of shortest paths Alpha can take up to that corner. Note that, except for the crossed out points, each number is the sum of the two numbers one to the left and one below. Proceeding this way, we find that there are 18 desired paths:



5. Richard is going up on an escalator. If he walks up with a constant speed, it takes him 30 seconds. If he walks down with the same constant speed, it takes him 120 seconds. How many seconds would it take him if he stays still on the escalator?

Answer: $\boxed{48}$

Solution: Divide the escalator into 240 imaginary steps. Then in 1 second, Richard would be going up by 2 steps when he is walking down and by 8 steps when he is walking up. Note that his speed going up when he stays still is the average of his speeds in the other two cases. So, if he stays still, he would be going up by $\dfrac{2+8}{2} = 5$ steps per second and it would take him $240 \div 5 = 48$ seconds.