Pi Math Contest - Euler Division
Sample Problems

1. Find the remainder when 2018 is divided by 11.

   Answer: $\boxed{5}$

   Solution: Through long division, we get

   $$2018 = 11 \times 183 + 5.$$

   So the answer is 5.

   Alternatively, note that 10 is 1 less than 11. This leads to even powers of 10 being 1 more than and odd powers of 10 being 1 less than a multiple of 11. So,

   $$2018 = 2 \times 10^3 + 0 \times 10^2 + 1 \times 10 + 8$$

   differs from $-2 + 0 - 1 + 8 = 5$ by a multiple of 11. Hence, the remainder is 5.

2. Andrew is 4 times as old as his sister Phoebe. In 2 years, he will be twice as old as Phoebe. How old was Andrew when Phoebe was born?

   Answer: $\boxed{3}$

   Solution: Let a denote Phoebe's current age. Then Andrew is currently $4a$. Note that the difference of their ages, $3a$, does not change over the years. In 2 years, Andrew will be twice as old as Phoebe. So, the difference of their ages (still $3a$) will be same as Phoebe's age at the time. Since, in 2 years, Phoebe grows from a to $3a$ years old, we find that $2a$ is 2 and a is 1. The answer is $3a$ which is 3.

3. In an acute triangle, the three angles form an arithmetic sequence. Also, one of the angles, in degrees, is a perfect cube. What is the difference of the largest and smallest angles in the triangle, in degrees?

   Answer: $\boxed{8}$

   Solution: The sum of the angles in any triangle is $180°$. If $\angle A < \angle B < \angle C$ form an arithmetic sequence, then $\angle B$ must be the average, which is $180°/3 = 60°$. The candidates for the degree of the perfect cube angle are $1, 8, 27, 64$, and 125. Note that 1, 8, 27, and 125 all give rise to an obtuse triangle. So, the perfect cube angle must be 64 degrees. This leads to the triple $56, 60, 64$. Hence, the answer is $64 - 56 = 8$.

4. Alpha Math Contest has 25 questions. Each correct answer is worth 10 points and each blank answer is worth 1 point. No points are given for incorrect answers. If Brandon scored 124 points, at least how many incorrect answers did he have?

Answer: $\boxed{0}$

Solution: Note that the points awarded from the correct answers is always a multiple of 10. Since Brandon's score ends in a 4, he must have had 4, 14, or 24 blank answers. With 4 blanks, he would need 12 correct to get 124 points, which leaves 9 incorrect. With 14 blanks, he would need 11 correct to get 124 points, which leaves 0 incorrect. With 24 blank answers, he can get at most 34 points, with 1 correct and rest blank but he scored way more than that. Hence, the smallest number of incorrect answers he could have made is 0.

Alternatively, noting the possibility of 11 correct, 14 blank, and no incorrect immediately shows that the answer is 0.

5. Alpha the Penguin will take a shortest path in a grid from $(0,0)$ to $(4,2)$ but she has to avoid a sea lion at $(2,1)$. How many different shortest paths can she take?

Answer: $\boxed{6}$

Solution: We present one of many possible solutions. Consider the following grid. Alpha is walking from lower left end to upper right end. The point that Alpha has to avoid is marked with an X. Note that Alpha has to visit exactly one of the points M or N on her way. There are 3 ways to go through M and 3 ways to go through N. Hence, the answer is 6.