Pi Math Contest Gauss Division

2024 Solutions

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Solutions

1. The expression

$$\frac{1}{\frac{1}{20}+\frac{1}{24}}$$

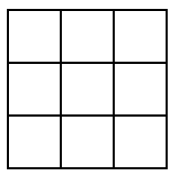
equals $\frac{a}{b}$ in simplest form, where a and b are relatively prime positive integers. What is $a+b$?

Answer (131): Observe that $\frac{1}{20}+\frac{1}{24}=\frac{6+5}{120}=\frac{11}{120}=\frac{b}{a}$, hence $a+b=131$.

2. If it takes 540 seconds to cut a log into 19 pieces, how many seconds are needed to cut it into 7 pieces at the same pace? The cutting is done one piece at a time.

Answer (180): To split a log into 19 pieces requires 18 cuts, and to get 7 pieces requires 6 cuts. If 18 cuts take 540 seconds, then 6 cuts would take 180 seconds.

3. A 3×3 metal grid, as shown below, is created using a total of 120 inches of metal wire. Determine the area of the square grid.



Answer (225): Let s be the side length of the metal grid, in inches. The total length of wire used is $8s$, accounting for the four horizontal and four vertical wires of length s each. Solving the equation $8s = 120$ yields $s = 15$. Consequently, the area of the square grid is $s^2 = 225$ square inches.

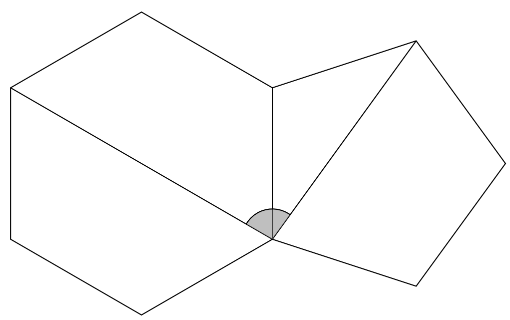
4. When the 4-digit number $\underline{PIMC}$ is divided by the 3-digit number $\underline{PIM}$, the quotient and the remainder add up to a perfect square. What is the value of C?

   Answer (006): Begin by noting the relationship:

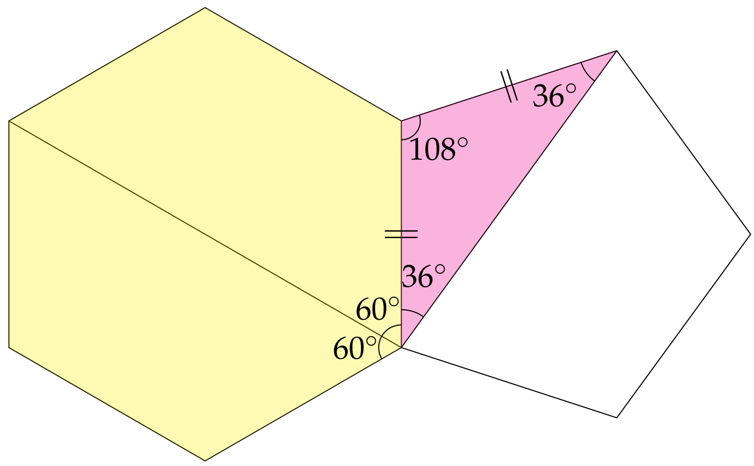
$$PIMC = 10 \cdot PIM + C.$$

   The sum of the quotient and the remainder, $10 + C$, is a perfect square. Given that C is a digit, the only valid solution is $C = 6$.

5. A regular hexagon and a regular pentagon, both with sides of the same length, are attached to each other along one of their sides, as shown below. Determine the degree measure of the angle indicated in the diagram.



   Answer (096): Recall that the angles of a regular hexagon and regular pentagon are 120° and 108°, respectively. The diagonal of the hexagon divides it into two congruent quadrilaterals (highlighted in yellow in the figure below), while the diagonal of the pentagon creates an isosceles triangle (shown in magenta in the figure below) and a quadrilateral. The hexagon contributes a 60° angle to the total, and the pentagon contributes 36° since the vertex angle of the isosceles magenta triangle is 108°.



6. Six friends, including Maxy and Lily, seat themselves in a row of six adjacent seats at the movies. How many arrangements are possible where there are an odd number of seats between Maxy and Lily?

   Answer (288): Number the seats from 1 to 6. There can be 1 or 3 seats between Maxy and Lily. Then there are 6 choices for the seats of Maxy and Lily: 1-3, 2-4, 3-5, 4-6, 1-5, or 2-6. We can order Maxy and Lily in 2 ways, and the remaining 4 people in $4! = 24$ ways. Thus, by the multiplication principle, the answer is $6 \cdot 2 \cdot 24 = 288$.

7. Twelve years ago, a father was three times as old as his son. Eight years from now, he will be twice as old as his son. What is the sum of the ages of the father and son now?

   Answer (104): Let F and S denote the ages of the father and son now. We are given $F - 12 = 3(S - 12)$ and $F + 8 = 2(S + 8)$. Combining these we get

$$F = 3S - 24 = 2S + 8.$$

   From these we derive $S = 24 + 8 = 32$ and $F = 2 \cdot 32 + 8 = 72$. The answer is $72 + 32 = 104$.

8. Let A, B, and C be distinct positive digits. When these digits are arranged to form the 3-digit numbers $\underline{ABC}$, $\underline{CAB}$, and $\underline{BCA}$, it is observed that $\underline{ABC}$ is divisible by 4, $\underline{CAB}$ is divisible by 5, and $\underline{BCA}$ is divisible by 9. What is the value of the largest of these 3-digit numbers?

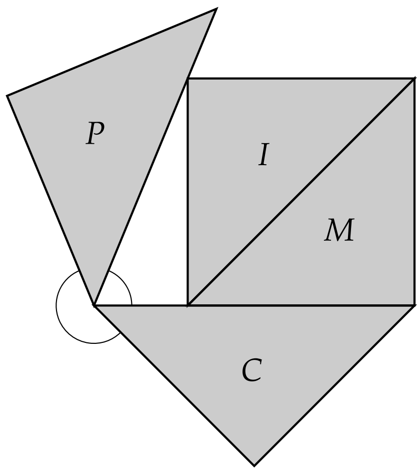
Answer (756): Since $\underline{CAB}$ is a multiple of 5, B must be 0 or 5. But $\underline{BCA}$ is a three-digit number, hence B cannot be 0. It follows that $B = 5$. Moreover, since $\underline{BCA}$ is a multiple of 9, the sum $A + 5 + C$ must also be a multiple of 9. This condition leads to two possibilities: either $A + C = 4$ or $A + C = 13$. In the former case, as A and C are distinct, the only valid combination is $A, C = 1, 3$; however, this fails to meet the requirement of $\underline{ABC}$ being a multiple of 4. Thus, we conclude that $A + C = 13$.

Considering the divisibility by 4, since ABC must be a multiple of 4, it follows that $\underline{5C}$ is also a multiple of 4. This condition implies that C can be either 2 or 6. Since $A + C = 13$ and A is a digit, it follows that $C = 6$. Consequently, we find that $A = 7$, $B = 5$, and $C = 6$, resulting in the largest three-digit number, which is 756.

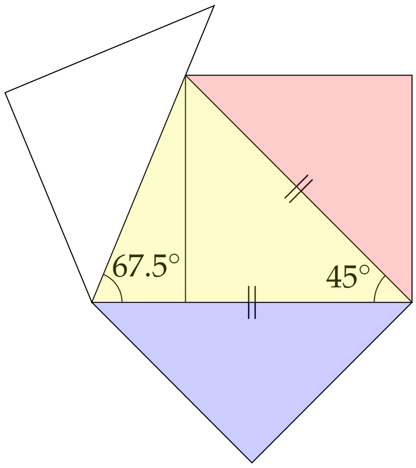
9. A family with two parents and six teenager children rides in an eight-seat SUV, featuring one row of two seats (including the driver's seat) and two rows with three seats each. One of the parents must occupy the driver's seat. Three of the children are triplets and wish to be seated in the same row. How many ways can the family seat themselves, subject to these two conditions?

Answer (576): There are 2 ways to choose the parent who sits in the driver's seat. Next, there are 2 ways to choose the row that the triplets sit in, followed by $3! = 6$ ways to seat the triplets in that row. Finally, there are $4! = 24$ ways to seat the remaining four people without restriction. The total number of ways is $2 \times 2 \times 6 \times 24 = 576$.

10. Let P, I, M and C be congruent isosceles right triangles arranged on a plane, as shown in the figure below. What is the degree measure of the positive difference between the larger and smaller angles between triangles P and C?



Answer (135): Draw the second diagonal of the square and construct the diagram as shown below.



As the blue and red triangles are congruent, the yellow triangle becomes an isosceles triangle with an angle at the vertex measuring 45 degrees. Consequently, the small angle between triangles P and C is 67.5 degrees, while the larger angle is calculated as $360 - 45 - 45 - 67.5 = 202.5$ degrees, resulting in a difference of $202.5 - 67.5 = 135$ degrees.

11. A cashier has an ample supply of \$1, \$5, \$10, \$20, and \$50 bills. For some positive integer d, it is impossible to make \$$d$ using fewer than ten of these bills. What is the smallest possible value of d?

    Answer (189): Intuitively, d should end in the digit 9 since five bills are required to make \$9 (one \$5 and four \$1), and we want \$$d$ to require ten bills. Therefore, we can find the smallest multiple of 10 that requires $10 - 5 = 5$ bills. We see that \$10 and \$20 can be made using one bill, while \$30 and \$40 require two bills. A quick check reveals that the smallest multiple of 10 that requires five bills is $\$180 = \$50 + \$50 + \$50 + \$20 + \$10$. Then $d = 180 + 9 = 189$.

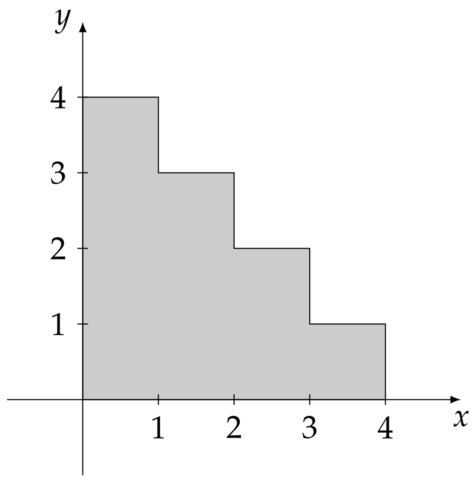
12. For how many of the first 1000 positive integers n is the non-negative difference between the decimal parts of $\frac{n}{2}$ and $\frac{n}{3}$ equal to $\frac{1}{6}$? (The *decimal part* of a number equals the part after the decimal point; for example, the decimal part of 3.5 is 0.5.)

    Answer (333): Let r be the remainder when we divide n by 6. Then the decimal parts of $n/2$ and $n/3$ are the same as the decimal parts of $r/2$ and $r/3$, respectively. Looking at each of the cases $r = 0, 1, 2, 3, 4$, and 5, we see that those decimal parts differ by $1/6$ exactly when $r = 1$ or 5. This gives us $167 + 166 = 333$ numbers.

13. What is the area, in square units, of the region in the xy-plane satisfying the inequality $\lfloor |x| \rfloor + \lfloor |y| \rfloor \leq 3$? The notations $|r|$ and $\lfloor r \rfloor$ respectively denote the absolute value of r and the greatest positive integer less than or equal to r.

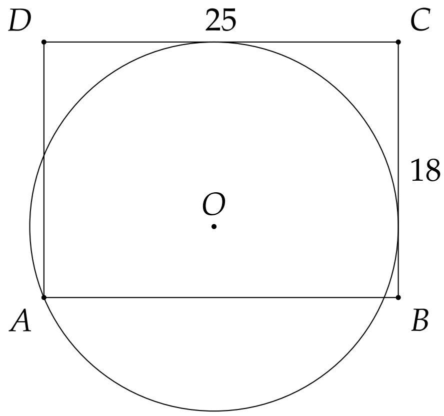
    Answer (040): Consider just the first quadrant. Since x and y are both surrounded by absolute values, it follows that if (x, y) satisfies the given inequality where $x, y \geq 0$, so do all of the points $(-x, y)$, $(-x, -y)$, and $(x, -y)$. Thus, we will find the area in the first quadrant determined by $\lfloor x \rfloor + \lfloor y \rfloor \leq 3$, then multiply our answer by 4.

    If $0 \leq x < 1$, then $\lfloor x \rfloor = 0$ and $\lfloor y \rfloor \leq 3$, so it follows that $0 \leq y < 4$ for this case. Similarly, if $1 \leq x < 2$, then $\lfloor x \rfloor = 1$ and $\lfloor y \rfloor \leq 2$, implying $0 \leq y < 3$. The graph of this inequality where $x, y \geq 0$ will resemble a staircase:



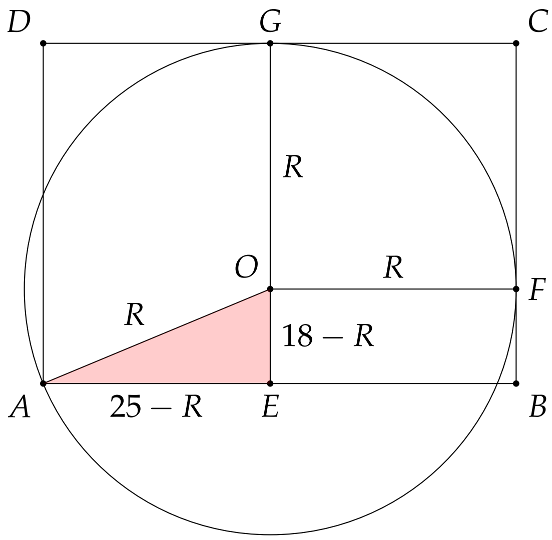
    The area of this region is 10 square units, so the total area is $10 \times 4 = 40$ square units.

14. In rectangle $ABCD$ below, $CB = 18$ and $CD = 25$. A circle centered at O passes through point A and is tangent to sides $\overline{CB}$ and $\overline{CD}$. What is OD^2?



Answer (313): Let perpendiculars dropped from O onto sides AB, BC, and CD intersect their respective sides at E, F, and G. Let R be the radius of the circle.

In this configuration, the quadrilateral $OFCG$ forms a square with side length R, and the triangle AOE is a right triangle with legs measuring $18 - R$, $25 - R$, and a hypotenuse of length R. Refer to the figure below.



Applying the Pythagorean theorem on $\triangle AOE$, we obtain the equation:

$$R^2 = (18 - R)^2 + (25 - R)^2.$$

Solving for R leads to the quadratic expression:

$$0 = R^2 - 86R + 949.$$

Factoring this expression results in:

$$(R-13)(R-73)=0.$$

Therefore, the potential values for R are 13 and 73. Since O lies within the rectangle, the valid solution is $R=13$.

Finally, using Pythagorean theorem on $\triangle ODG$, we get

$$OD^2 = OG^2 + DG^2 = R^2 + (25-R)^2 = 13^2 + 12^2 = 313.$$

15. Nine chess players participate in a tournament and are ranked from 1 to 9. Given that Elizabeth placed 5th, Anna placed ahead of Benjamin, Benjamin placed ahead of Carly, Danielle placed ahead of Elizabeth, Elizabeth placed ahead of Fiona, and Gary placed ahead of both Harold and Isaac, how many possible rankings are there?

    Answer (canceled): Let us denote the rankings as $A-I$ using the initials of the players. We know that $E=5$,

    $$A < B < C$$

    $$D < 5 < F$$

    $$G < H,\ G < I$$

    First let's look at $D < 5 < F$. It gives $4 \cdot 4 = 16$ possibilities for the ranks of D and F. That leaves 6 players. Those can be ranked in 6! ways among themselves. However, due to the restriction $A < B < C$ we should divide it by 6 because it happens 1/6 of the time, and $G < H$ and $G < I$ means G is ranked first among G, H, I which happens 1/3 of the time. So, those 6 players can be ranked in $\frac{6!}{6 \cdot 3} = 40$ ways. Using the multiplication principle, we get $16 \cdot 40 = 640$ possible rankings.

16. Three runners compete in a 400-meter race. When the first runner crosses the finish line, the second and third runners are 10 and 49 meters behind the finish line, respectively. When the second runner finishes the race, how many meters will the third runner be behind the finish line? Assume each runner runs at a constant speed.

    Answer (040): When the first runner finishes, the second and third runners have run 390 meters and 351 meters, respectively. Note that $\frac{390}{351} = \frac{10}{9}$, so their speeds are in ratio of 10 to 9. Hence, when the second runner runs 400 meters, the third runner runs 360 meters, and the difference between them is 40 meters.

Remark: One can write the solution above more formally as below:

Let t represent the time it takes for the first runner to finish the race. This leads to the following equations:

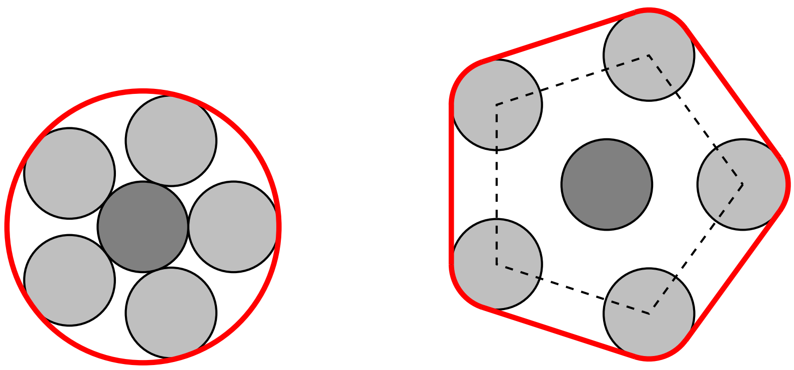
$$v_1 t = 400,$$
$$v_2 t = 390,$$
$$v_3 t = 351,$$

where v_i denotes the speed of the ith runner. Now, let T be the time at which the second runner completes the race. On one hand, we have $v_2 T = 400$. On the other hand, utilizing the previously derived results, we find:

$$\frac{400}{v_3 T} = \frac{v_2 T}{v_3 T} = \frac{v_2}{v_3} = \frac{390}{351} = \frac{10}{9}.$$

As a result, we determine that $v_3 T = 360$, representing the distance covered by the third runner when the second runner finishes the race. The total distance covered by both runners is 40.

17. Six identical hockey pucks, each with a radius of 1 inch, are arranged as shown on the left below. Five of these pucks are equally spaced and touch the center puck. A circular rubber band is then placed around the outer five pucks. Upon moving the five outer pucks away from the center puck, the rubber band doubles in length as shown on the right below. Let P be the perimeter of the pentagon whose vertices are the centers of the pucks, as shown on the right below. What is the integer closest to P?



Answer (031): The rubber band has a radius of 3, resulting in a perimeter of 6π. Upon doubling in size during the second phase, the overall length of the stretched rubber band becomes 12π. Notably, this stretched length is equivalent to the combined perimeter of the desired pentagon and a full circle with a radius of 1. This can be observed by considering that the angles formed by the circular portions of the stretched band sum to 360 degrees, with each circle having a radius of 1. Consequently, the perimeter of the pentagon is determined to be $12\pi - 2\pi = 10\pi$, approximately equal to 31.4, and rounding to the nearest integer results in 31.

18. Amelia and Brandon take turns taking candies from a jar initially containing N candies: Amelia takes 1 candy, Brandon takes 2, Amelia takes 3, and so on, where the next person always takes one more candy. If there aren't enough candies in the jar, that person takes all remaining candies. If Amelia takes 2024 candies altogether, what are the last three digits of N?

    Answer (004): Note that Amelia accumulates

    $$1 + 3 + \cdots + (2n - 1) = n^2$$

    candies during her first n turns, provided there are still candies available. Since $2024 = 44^2 + 88$, Brandon draws candies for 44 turns. The total number of candies he obtains is given by the sum $2 + 4 + \cdots + 88$, which can be expressed as

    $$2 \cdot \frac{44 \cdot 45}{2} = 1980.$$

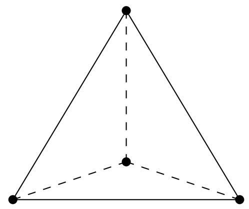
    Consequently, the initial number of candies in the jar is calculated as $N = 1980 + 2024 = 4004$, with its last three digits being 004.

19. How many distinct ways can the set $S = \{1, 2, 4, 8, \ldots, 2^9\}$ be partitioned into two disjoint, non-empty subsets A and B so that $A \cup B = S$, and the sum of the elements in A is a multiple of the sum of the elements in B? One example is $A = \{4, 8, \ldots, 2^9\}$ and $B = \{1, 2\}$.

    Answer (007): The sum of the elements in S is $2^{10} - 1 = 1023$. Suppose the sum of the elements of B is b, so that the sum of the elements of A is $1023 - b$. Then $1023 - b$ needs to be a multiple of b, which occurs if and only if 1023 is a multiple of b. Moreover, we require $1023 - b \geq b$, or $b \leq 511$.

    It follows that the candidate values for b are the proper divisors of 1023. The prime factorization of 1023 is $3^1 \times 11^1 \times 31^1$, so 1023 has $(1+1)(1+1)(1+1) - 1 = 7$ proper divisors, the largest of which is $\frac{1023}{3} = 341 < 511$. To show that all seven candidate values for b are indeed possible, we recall that every positive integer has a unique binary (base 2) representation. This means that for every b, there exists a subset B of S whose sum is b. Therefore the number of ways is 7.

20. Four ants are initially located at different vertices of a regular tetrahedron. Each ant randomly moves to one of the three adjacent vertices. If the probability that each ant ends up at a different vertex is $\frac{m}{n}$ where m and n are relatively prime integers, what is $m+n$?



Answer (010): Let's designate the ants as Ant 1, Ant 2, Ant 3, and Ant 4. Ant 1 has three possible destinations. Assuming Ant 1 moves to point A, there are two scenarios: either Ant A moves to 1, while the remaining two ants swap places, or Ant A moves to B, causing Ant B to move to C, and Ant C to move to 1. Each of these scenarios can occur in 1 and 2 different ways, respectively. Consequently, there are a total of $3 \cdot (1+2) = 9$ ways in which the ants can end up in different locations.

Considering all possible movements of the four ants, with each ant having three available places to go, there are a total of 3^4 different ways in which the ants can move. Therefore, the desired probability is calculated as $\frac{9}{3^4} = \frac{1}{9}$, resulting in a final sum of $1+9=10$.

21. Olivia selects a quadratic polynomial. She multiplies the quadratic polynomial by two different linear polynomials and displays the resulting products but erases the constant terms in both products. When Elijah enters the room, she sees the two expressions:

$$x^3 + 2x^2 - x$$

$$x^3 - x^2 - 4x$$

Despite the missing constant terms, Elijah can correctly determine the sum of the squares of the roots of Olivia's original quadratic polynomial. What is this sum?

Answer (005): Let P represent the quadratic polynomial. Notably, P divides both results. The Euclidean algorithm implies that P must also divide the difference, which takes the form $3x^2 + 3x + \star$. For convenience, let's assume that P has a leading coefficient of 1, as it does not affect the sum of its roots. Consequently, we can express $P(x)$ as $x^2 + x + C$ for some real number C.

Substituting this expression into the first equation, we solve for

$$x^3 + 2x^2 - x + M = (x^2 + x + C)(x + D).$$

Equating the coefficients of x^2 on both sides yields $D = 1$, and equating the coefficients of x results in $C = -2$. Thus,

$$P(x) = x^2 + x - 2 = (x - 1)(x + 2),$$

and the square sum of the roots is $1^2 + (-2)^2 = 5$.

22. How many integer pairs (a, b) are there such that $1 \le a \le 1000$, $1 \le b \le 1000$, and $\text{lcm}(a, b) = 30\,\text{gcd}(a, b)$?

Answer (730): Let $a = g \cdot a_1$ and $b = g \cdot b_1$, where $g = gcd(a, b)$. Using

$$\text{lcm}(a, b) = \frac{ab}{g} = ga_1b_1 = 30g$$

we get $a_1b_1 = 30$. Since a_1 and b_1 are relatively prime positive integers, there are 8 solutions for (a_1, b_1):

$$(1, 30),\ (2, 15),\ (3, 10),\ (5, 6),$$

and the reverse of these.

For each of these pairs, we find the number of possible g values so that a_1g and b_1g are both less than or equal to 1000. That gives us the number of (a, b) pairs in each case. We deduce that there are

$$2(33 + 66 + 100 + 166) = 2 \cdot 365 = 730$$

solutions for (a, b).

23. Isabella rolls a fair six-sided dice repeatedly until she rolls every number at least once. What is the closest integer to the expected number of rolls needed?

Answer (015): Let E_n represent the expected number of throws required until all six outcomes have occurred, given that exactly n distinct outcomes have been observed. Our objective is to determine E_0. It is evident that when $n = 6$, $E_6 = 0$.

The recurrence relation for E_n is given by:

$$E_n = \frac{n}{6}(E_n + 1) + \frac{6 - n}{6}(E_{n+1} + 1) = \frac{n}{6}E_n + \frac{6 - n}{6}E_{n+1} + 1.$$

Rearranging this equation, we obtain:

$$E_n - E_{n+1} = \frac{6}{6-n}.$$

By setting $n = 0, 1, \ldots, 5$ in the above equation, summing them up, and considering that $E_6 = 0$, we derive:

$$\begin{aligned} E_0 &= E_0 - E_6 \\ &= (E_0 - E_1) + (E_1 - E_2) + \cdots + (E_5 - E_6) \\ &= \frac{6}{6} + \frac{6}{5} + \cdots + \frac{6}{1} \\ &= 1 + 1.2 + 1.5 + 2 + 3 + 6 \\ &= 14.7. \end{aligned}$$

Rounding to the nearest integer, the result is 15.

24. A real number $3 \le r \le 1000$ is chosen, and a sequence is constructed by the rules $a_0 = r$ and $a_n = \sqrt{15a_{n-1} - 36}$ for $n \ge 1$. What is the sum of all possible values of $\lfloor a_{2024} \rfloor$? The notation $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

Answer (075): Let's define the function $f(r)$ as

$$f(r) = \sqrt{-36 + 15\sqrt{-36 + 15\sqrt{\ldots \sqrt{r}}}}$$

where there are 2024 square roots. Then the possible values of values of $\lfloor a_{2024} \rfloor$ are all possible values of $\lfloor f(r) \rfloor$ for $3 \le r \le 1000$.

It can be verified that if $a_0 = 3$, then $a_n = 3$ for all positive integer n. Similarly, if $a_0 = 12$, then $a_n = 12$ for all positive integers n. Hence, $f(3) = 3$ and $f(12) = 12$.

Since $f(r)$ is continuous with $f(3) = 3$ and $f(12) = 12$, intermediate value theorem implies that $f(r)$ attains all integer values between 3 and 12, inclusive.

Next, we prove that $f(r) < 13$ when $r \le 1000$. Observe that

$$a_n = \sqrt{15a_{n-1} - 36} < 4\sqrt{a_{n-1}}.$$

Since a_n is increasing in r for all n we obtain $a_0 \le 1000$, $a_1 \le 4\sqrt{1000} < 127$, $a_2 < 4\sqrt{127} < 46$, $a_3 < 4\sqrt{46} < 28$, $a_4 < 4\sqrt{28} < 22$, $a_5 < 4\sqrt{22} < 19$, $a_6 < 4\sqrt{19} < 18$, $a_7 < 4\sqrt{18} < 17$ for all $3 \le r \le 1000$.

Hence, $a_8 < \sqrt{15 \cdot 17 - 36} < 15$, $a_9 < \sqrt{15 \cdot 15 - 36} < 14$, $a_{10} < \sqrt{15 \cdot 14 - 36} < 13.2$, $a_{11} < \sqrt{15 \cdot 13.2 - 36} < 13$.

Next, observe that if $a_{n-1} < 13$ then

$$a_n = \sqrt{15a_{n-1} - 36} < \sqrt{15 \cdot 13 - 36} < 13.$$

Hence, $a_{2024} < 13$ and therefore, $f(r) < 13$.

We conclude that all possible values for $\lfloor a_{2024} \rfloor$ are $3, 4, \cdots, 12$ and they sum to

$$3 + 4 + \cdots + 12 = 75.$$

25. Sophia has a collection of rocks with possibly different weights. She discovers that she can distribute these rocks into 3 piles of equal weight, 4 piles of equal weight, and also 5 piles of equal weight. What is the minimum positive number of rocks in her collection for which this is possible?

Answer (009): First observe that there must be a minimum of 8 rocks in the pile. The rationale is when fewer than 8 rocks are divided into 4 piles, at least one pile contains exactly one rock. This rock weighs $\frac{W}{4}$ units, where W is the total weight of the rocks. However, it then becomes impossible to divide the rocks into 5 piles of equal weight because the weight of the pile containing the rock weighing $\frac{W}{4}$ exceeds $\frac{W}{5}$.

Next, we show that this is impossible to do with 8 rocks. To demonstrate this, let's assume without loss of generality that the total weight of the rocks is 60. Observe that

a) No rock has a weight more than 12 units, since each rock belongs to a pile of weight 12 units when rocks are divided into 5.

b) When rocks are divided into 4 piles of equal weight, each pile has exactly two rocks. This is because previous observation implies each pile has at least 2 rocks, and total number of rocks implies each pile contains exactly 2 rocks.

When divided into 5 piles, it becomes apparent that at least two of the piles contain only a single rock. Consequently, there are two rocks with a weight of 12. Hence, there are two rocks of weight 3 due to (b).

When rocks are divided into 3 groups, two rocks weighing 12 must be in separate piles. Let's make casework on the number of the rocks on the third pile. If this number is n, then it is at most 4 since each 12 must be accompanied by at least one rock each, and it is at least two due to (a).

- When $n = 4$, two 12's must be grouped with exactly one rock of weight 8 each. When divided into four piles, 12, 12, 8, 8 must belong to separate piles, and due to (b) the remaining rocks have weight of 7 units each. However, these weights make it impossible to split the rocks into 5 piles of equal weight.
- When $n = 3$, two of the 3's cannot belong to the third pile due to (b). The rocks weighing 12 must be accompanied by one and two rocks. Hence, one 12 is grouped with an 8 and the other 12 is grouped with a 3 and a 5. The third group consists of rocks weighing 3, x, and y units. When rocks are divided into four, two 12's and 8 are in separate piles. Due to (b), one of the rocks weigh 7, and hence $\{x, y\} = \{7, 10\}$. However, now it is impossible to divide rocks into 5 piles of equal weight.
- When $n = 2$, the third pile cannot contain a rock with weight 3 due to (b). We check the sub cases when 3's are grouped together or separate.
  - If they are together, their group must contain a rock of weight 12, and at least one more rock. The other 12 must be grouped with at least one rock. The only case that works is one group is (12,3,3,2), the other group is (12,8) and the third group is (x, y). Two 12's and 8 are in separate groups when rocks are divided into 4. Due to (b), there must be one rock with weight 7, and this rock belongs to the third group. The other rock in this group weighs 13 units, which contradicts with (a).
  - If they are separate, then each 12 in grouped with a 3. The only way this works is when the groups are (3,5,12), (3,5,12), and (x, y). Due to (b) the only possibility for $\{x, y\}$ is $\{10, 10\}$. However, these weights are impossible to divide into 5 groups of equal weight.

Finally, we demonstrate an example that works for 9 rocks: Let the weights of the rocks be as

$$\{3, 4, 5, 6, 6, 7, 8, 9, 12\}.$$

These can be distributed into 5 piles as

$$(12), (3, 9), (4, 8), (5, 7), (6, 6),$$

into 4 piles as

$$(3, 12), (6, 9), (7, 8), (4, 5, 6),$$

and into 3 piles as

$$(8, 12), (5, 6, 9), (3, 4, 6, 7).$$