

Pi Math Contest Fermat Division

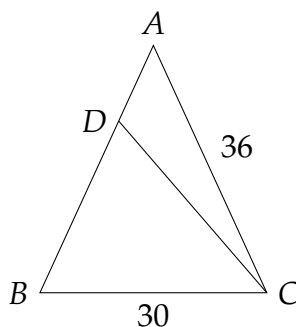
2024

INSTRUCTIONS

1. DO **NOT** OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU!
2. This is a 25 question test. Each question has a two digit answer: 00, 01, 02, 03, . . . ,99. For example, if the answer is 1, you must bubble 01 instead of 1.
3. Mark your answer to each question on the Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 10 points for each correct answer, 1 point for each problem left unanswered, and 0 points for each incorrect answer.
5. Only pencils, pencil sharpeners, erasers, and blank scratch papers are allowed. All other aids, including but not limited to calculators and notes, are not allowed.
6. Figures are not necessarily drawn to scale.
7. When your proctor gives the signal, begin working on the problems. You will have **40 minutes** to complete the test.
8. After the exam, return your **Answer Form, Test Booklet and scratch papers** to your proctor.
9. Do NOT discuss any exam questions until **March 18th**, after which problems and solutions will be available on the contest website.

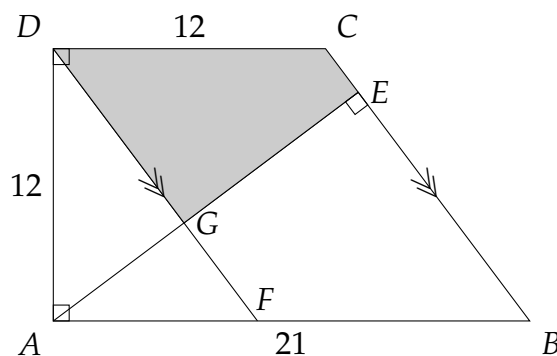
1. Allie purchases three t-shirts, priced at \$13.50 each. Dana purchases two jackets, priced at \$27.75 each. How many more dollars does Dana spend than Allie?
2. What integer is equal to 36% of 75?
3. A recipe for 18 chocolate chip cookies calls for $1\frac{2}{3}$ cups of flour. If Abby has 5 cups of flour and an ample supply of every other ingredient, what is the greatest number of chocolate chip cookies Abby can make?
4. Hector is six years older than Marisol, Marisol is 4 years older than Cielo, and Hector is twice as old as Cielo. What is the sum of their ages?
5. The greatest common divisor of n and 180 is 6. Given that n is a positive integer less than 100, what is the greatest possible value of n ?
6. Ten people, including Robert and Saul, randomly seat themselves at a round table containing ten chairs. The probability that Robert and Saul do **not** sit next to each other is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?
7. Let x and y be positive integers such that $xy + x + y = 220$. What is $x + y$?
8. Parallelogram $ABCD$ has vertices $A(1, 1)$, $B(2, 5)$, $C(a, b)$ and $D(7, 3)$ in the xy -plane. What is $a + b$?
9. Eric and Joshua are sitting near a farm containing 2-legged chickens and 4-legged cows, and no other animals. Eric counts 102 legs, and Joshua counts 39 animals. How many chickens are on the farm?

10. Consider the statement "If a positive 2-digit integer is not divisible by 2, 3, or 5, then it is prime." How many positive 2-digit integers make this statement false?
11. Isosceles triangle ABC has $m\angle ABC = m\angle ACB$. Point D is on side AB so that $m\angle BDC = m\angle ABC$. Given that $AC = 36$ and $BC = 30$, what is the perimeter of triangle ACD ?



12. The positive integer a has exactly 5 positive factors, and the positive integer b has exactly 7 positive factors. The integer $a \times b$ must either have m positive factors or n positive factors, where $m \neq n$. What is $m + n$?
13. An equilateral triangle with side length 4 cm is drawn on the plane. Let S be the set of points in the plane that are outside the triangle, but within 2 cm of some point on or inside the triangle. The area of S equals $a\pi + b$ cm^2 for some positive integers a and b . What is $a + b$?
14. Eight rectangles with dimensions 2×8 , 3×3 , 3×6 , 3×8 , 3×12 , 4×14 , 5×6 , and 6×11 are arranged to form a larger rectangle, without any overlap or gaps. What is the perimeter of the larger rectangle?

15. David cuts a 4-inch cube into 64 unit cubes, each with side length 1 inch. Half of these cubes are painted red on all faces, and the rest are painted blue on all faces. When David reassembles the unit cubes to form a 4-inch cube, what is the greatest possible surface area of this cube, in square inches, which is painted red?
16. For how many integers n between 1 and 100, inclusive, is the product $n(n+1)(n+2)$ divisible by 15?
17. Trapezoid $ABCD$ has $AB \parallel CD$, and $m\angle CDA = m\angle DAB = 90^\circ$. Point E is on segment BC so that $AE \perp BC$, and point F is on AB so that $DF \parallel BC$. Segments AE and DF intersect at G . Given that $AB = 21$ and $CD = AD = 12$, what integer is closest to the area of trapezoid $CDGE$?



18. Alice and Brenda each arrive at a diner at a randomly chosen time between 2:00 pm and 3:00 pm, independently of each other. While Alice is patient and willing to wait until Brenda arrives, Brenda will only wait for up to 15 minutes for Alice to arrive, before leaving. The probability that Alice and Brenda meet each other at the diner equals $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

19. There are four positive integers less than 100 whose squares end in the digits "16." One such integer is 4, as $4^2 = 16$. What are the last two digits of the sum of the other three integers?
20. Ann, Beth, and Cia split a large basket containing 100 apples, 100 bananas, and 100 oranges so that each person received 100 pieces of fruit. Ann received one more apple than Beth, and Beth received two more bananas than Cia. Given that Cia received 7 apples, what is the **greatest** whole number of oranges Cia could have received?
21. Let a , b , and c be integers such that

$$\begin{aligned}a + b + c &= 1, \\4a + 2b + c &= 8, \text{ and} \\9a + 3b + c &= 27.\end{aligned}$$

What is the value of $16a + 4b + c$?

22. Hannah rolled six standard 12-sided dice (with faces numbered 1 through 12). The product of her dice rolls is 178,200, and no two of Hannah's rolls showed the same number. What is the sum of Hannah's dice rolls?
23. 100 people sit at a round table, and every person is either a knight (who always tells the truth), or a knave (who always lies). When every person at the table was asked, "Of the two people sitting next to you, how many of them are knights?", fifty people answered "0," and fifty people answered "2." What is the greatest possible number of knaves who are sitting at the round table?
24. How many non-empty subsets S of the set $\{2, 3, 5, 7, 11, 13, 17, 19\}$ are there such that the sum of the elements of S is divisible by 4? One example is $S = \{2, 7, 11\}$.

25. A function $f(x)$ satisfies the following properties for all positive integers x :

$$f(x)^2 = f(x^2) + 2, \text{ and}$$

$$f(x) = 2f(2x) - 3x.$$

The sum of all possible values of $f(8)$ equals $\frac{m}{n}$, where m and n are relatively prime positive integers. What are the last two digits of $m + n$?