Pi Math Contest Fermat Division

2022

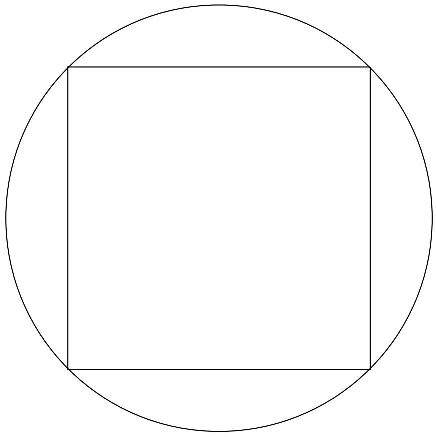
1. Find the value of $2 + 0 \times 2 - 2$.

2. Compute
$$\frac{333 - 33 + 3 \times 33}{3^3 - 3 - 3}.$$

3. How many ways are there to select one marble from a bag of 20 unique red marbles, 29 unique blue marbles, and 38 unique green marbles?

4. How many total sides are there among 15 triangles and 4 squares, if no side is shared by more than one shape?

5. A square is inscribed in a circle, as shown below. If the area of the circle is 16π, what is the area of the square?



6. Four times a number plus three is equal to seven times the number minus twelve. What is the number?

7. Five friends go to a local grocery store to buy bottles of water. Each bottle of water costs \$1.50, and each person has \$10.00. If they combine their money, what is the maximum number of bottles they can buy?

8. Bamboo grows at a rate of 1.7 inches per day. Tanya buys a bamboo stalk which currently stands 1 foot and 3 inches tall. If Tanya is 5 feet and 6 inches tall, then in how many days will the bamboo stalk grow to her current height? Recall that 1 foot equals 12 inches.

9. Compute the sum

$$\gcd(1,12)+\gcd(2,12)+\cdots+\gcd(11,12)+\gcd(12,12),$$

where $\gcd(a,b)$ denotes the greatest common divisor of a and b.

10. Annie and Mercie each have 32 pencils. To start, Annie gives Mercie half of her pencils. Since they are overly kind to each other, the person who has more pencils will always give half of her pencils to the other. This continues until the person with more pencils has an odd number of pencils. At that point, how many pencils will Mercie have?

11. Simplify:
$$\frac{2^{2^{0^2}} + 2^{2^0} + 2^2 + 2}{2^{0^{2^2}} + 2^{0^2} + 2^0 + 2}.$$

12. Let N be the number of three-digit positive integers that have at least two distinct digits. Find the last two digits of N.

13. A square and a regular hexagon have equal perimeters. If the area of the hexagon is $24\sqrt{3}$, what is the area of the square?

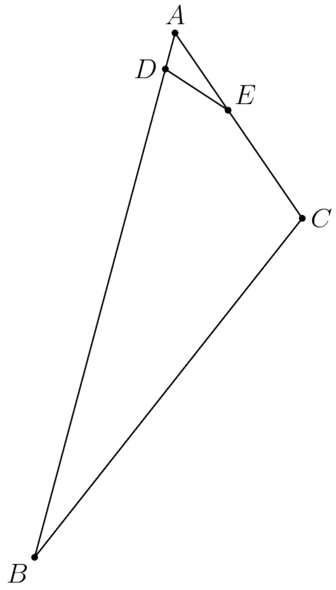
14. Suppose 80% of A is 56% of B, and 45% of B is 27% of C. What percent of C is 150% of A?

15. Every day, a sprinkler first turns on at 9 AM. To save water, Alice turns it off every 50 minutes, at 9:50 AM, 10:40 AM, and so on. To water the plants, Bob turns it on every 60 minutes, at 10 AM, 11 AM, and so on. They continue playing like this until 2 PM. At 2 PM, the sprinkler turns off for the rest of the day. The fraction of the 24 hours of the day the sprinkler is on can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

16. Three different primes have a mean of 58. What is the largest possible value of the median of these three primes?

17. A two-digit number is 33 more than a positive perfect square and 31 less than another positive perfect square. What is it?

18. In the following diagram, $AD = 2, AE = 5, DE = 4$, and $EC = 7$. Also, $\angle AED = \angle ABC$. What is the sum of the lengths BD and BC?



19. Each square is replaced with an addition symbol $(+)$ or a multiplication symbol $(\times)$ in the expression

$$1\square 2\square 3\square 4\square 5.$$

If the greatest possible result when the arithmetic operations are carried out is M and the least possible result is m, find the remainder when $M - m$ is divided by 100.

20. Ali wants to distribute 13 identical candies among Mason, Hanna, Tiger, and Jonathan, while satisfying all of the following conditions:

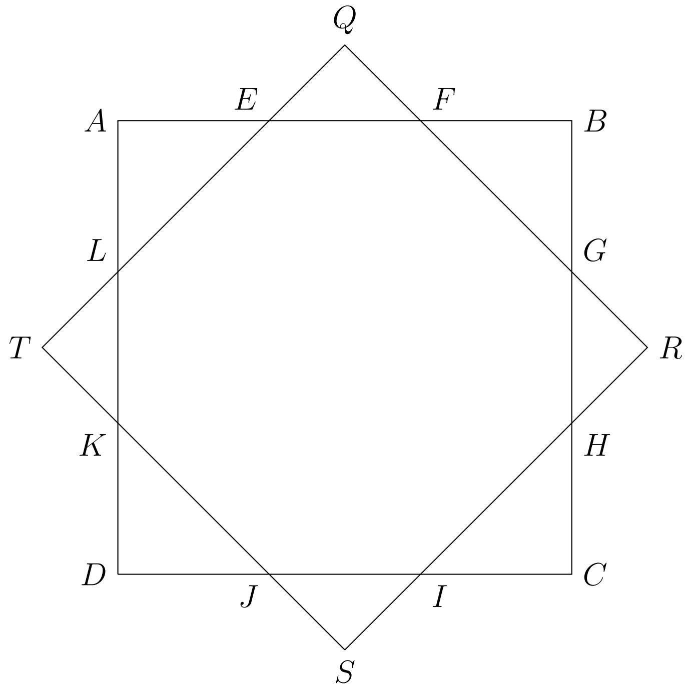
- Mason will get an odd number of candies.
- Hanna will get more candies than Mason.
- Tiger will get more candies than Mason and Hanna combined.
- Jonathan will get more candies than Tiger.

How many ways are there for Ali to distribute the candies?

21. Compute the sum of the distinct prime factors of

$$2^{15} + 2^8 - 2^7 - 1.$$

22. In the diagram below, $ABCD$ is a unit square. E, F, G, H, I, J, K, L divide each side of the square into three equal segments, and lie on square $QRST$. The ratio of the area of the intersection of these two squares to the area of the union of these two squares is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



23. Let N be the number of arrangements of the six letters $ABCDEF$ such that if two letters are in the same row in the diagram below, they cannot be adjacent in the arrangement. For example, $EACFBD$ is a valid arrangement while $ADFBEC$ is not, because D and F are adjacent but both are in the same row in the diagram. Find the last two digits of N.

A

B C

D E F

24. A circular target at an archery range has a radius of 11 units, and consists of concentric circles of radii 1, 3, 5, 7, 9, and 11. The target's areas are painted red and white in an alternating fashion, starting with red in the middle circle. Bernadetta shoots two arrows at the target, and each arrow hits the target at a uniformly random position; the first arrow hits a red region, and the second arrow hits a white region. The probability that the first arrow is closer to the center of the target than the second arrow is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

25. Find the remainder when

$$\left\lfloor\sqrt{1}\right\rfloor + \left\lfloor\sqrt{2}\right\rfloor + \left\lfloor\sqrt{3}\right\rfloor + \cdots + \left\lfloor\sqrt{2021}\right\rfloor + \left\lfloor\sqrt{2022}\right\rfloor$$

is divided by 100, where $\lfloor x\rfloor$ denotes the greatest integer less than or equal to x.